type theory

Γ ⊢ A type
Γ ⊢ A = B type
Γ ⊢ M : A
Γ ⊢ M = N : A
cubical type theory

formal intervals \( I \)

\[
\begin{align*}
\Gamma &\vdash 0 : I \\
\Gamma &\vdash 1 : I \\
x : I &\in \Gamma \\
\Gamma &\vdash x : I \\
\Gamma &\vdash r : I \\
\Gamma &\vdash s : I \\
\Gamma &\vdash r \land s : I \\
\Gamma &\vdash \neg r : I \\
\Gamma &\vdash r \lor s : I
\end{align*}
\]
cubical type theory

formal intervals $\mathbb{I}$

$x_1 : \mathbb{I}, x_2 : \mathbb{I}, ..., x_n : \mathbb{I} \vdash M : A$

$\iff M \text{ is an } n\text{-cube in } A$

$M\langle 0/x \rangle \quad M\langle 1/x \rangle \quad M\langle y/x \rangle$
cubical type theory

formal intervals $\mathbb{I}$

ordinary typing rules hold uniformly

$$\Gamma, a:A \vdash M : B$$

$$\Gamma \vdash \lambda a.M : (a:A) \to B$$

with any number of $\mathbb{I}$ in the $\Gamma$
cubical type theory

formal intervals $\mathbb{I}$

ordinary typing rules hold uniformly

\[
\Gamma, a:A \vdash M : B \\
\Gamma \vdash \lambda a.M : (a:A) \rightarrow B
\]

with any number of $\mathbb{I}$ in the $\Gamma$

function extensionality due to dimensions commuting with function application
cubical type theory

formal intervals \( \mathbb{I} \)

canonicity

any closed term of \( \mathbb{N} \) is equal to some numeral

type-theory tango: internalization of judgmental structure, harmony
cubical type theories

base category

structural rules + operators \( \{0, 1, \land, \lor, \neg, \ldots\} \)

most developed: cartesian, de morgan
cubical type theories

base category

structural rules + operators \{0,1,\land,\lor,\neg,...\}
most developed: cartesian, de morgan

kan structure

cofibrations, fiberwise fibrant replacement
**Cubical type theories**

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<th>Base category</th>
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RedPRL

RedPRL

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RedPRL
redtt specialities

higher inductive types

two-level type theory

nbe-like algorithm (conjectured correct)

extension types

judgmental refinements

holes, tactics, unification
redtt specialities

higher inductive types

two-level type theory

nbe-like algorithm (conjectured correct)

extension types

judgmental refinements

holes, tactics, unification

see demo
redtt specialities

higher inductive types

a general schema; indexed ones on the way

see chtt part 4 [Cavallo & Harper]
redtt specialities

two-level type theory

(no equality types yet)
Redtt specialities

todo: many-level type theory

discrete: paths equal to equality consistent with (strict) UIP
reddtt specialities

nbe algorithm

cubicaltt adopts a similar one
redtt specialities

nbe algorithm

cubicaltt adopts a similar one

difficulty 1: value re-evaluation: loop$_x[0/x]$

difficulty 2: constraints: r=s
redtt specialities

nbe algorithm

cubicaltt adopts a similar one

difficulty 1: value re-evaluation: $\text{loop}_x[0/x]$

difficulty 2: constraints: $r=s$

decidable: $\Phi \models r = s$
todo

correctness of nbe
todo

correctness of nbe

equality types
todo

correctness of nbe

equality types

user-defined tactic, pattern matching, etc
todo

correctness of nbe

equality types

user-defined tactic, pattern matching, etc

improved kan operations of universes
todo

correctness of nbe

equality types

user-defined tactic, pattern matching, etc

improved kan operations of universes

synthetic homotopy theory (!)
synthetic homotopy theory

dotted line:

todo

RedPRL

"obvious"

standard homotopy theory

Redtt