Covering Spaces in Homotopy Type Theory

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Homotopy type theory (HoTT) is an exciting new interpretation of intensional type theory which provides a synthetic framework for homotopy theory. It is natural to ask whether we can restate various homotopy-invariant concepts and theorems from classical homotopy theory. In this talk I will explore one fundamental construct: *covering spaces*. It turns out that we can express covering spaces (up to homotopy) elegantly in HoTT as follows.

Definition 1. A *covering space* of a type (space) A is a family of sets indexed by A.

That is, the type of a covering of A is simply $A \to \mathsf{Set}$ where Set is the type of all sets. To verify that this formulation matches the classical one, I proved in HoTT several expected properties of covering spaces, including that covering spaces of a pointed, path-connected A are classified by functors $\pi_1(A) \to \mathsf{Set}$ where $\pi_1(A)$ is the fundamental group of A, that homotopy-equivalent classes of paths with one end fixed form a universal covering space, and that simply-connectedness implies universality.

I will review the key ideas in the proofs. Some interesting techniques employed in the current proofs seem applicable to other constructions as well. It is worth emphasizing that every proof mentioned in this talk has been fully mechanized [1] in the proof assistant Agda. The code of critical parts will be demonstrated during the talk.

Looking forward, I am working on the generalization of the results from sets to groupoids. These attempts will also be discussed if time permits.

References

[1] HoTT library in Agda. https://github.com/HoTT/HoTT-Agda.

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