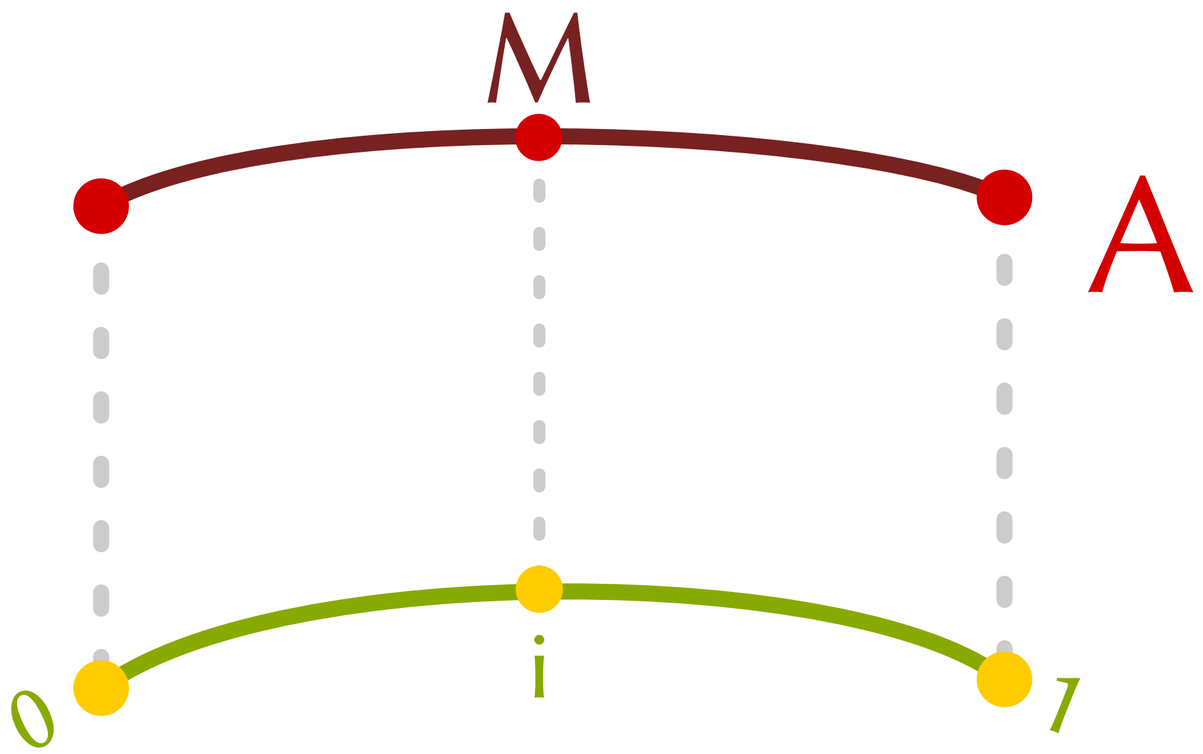
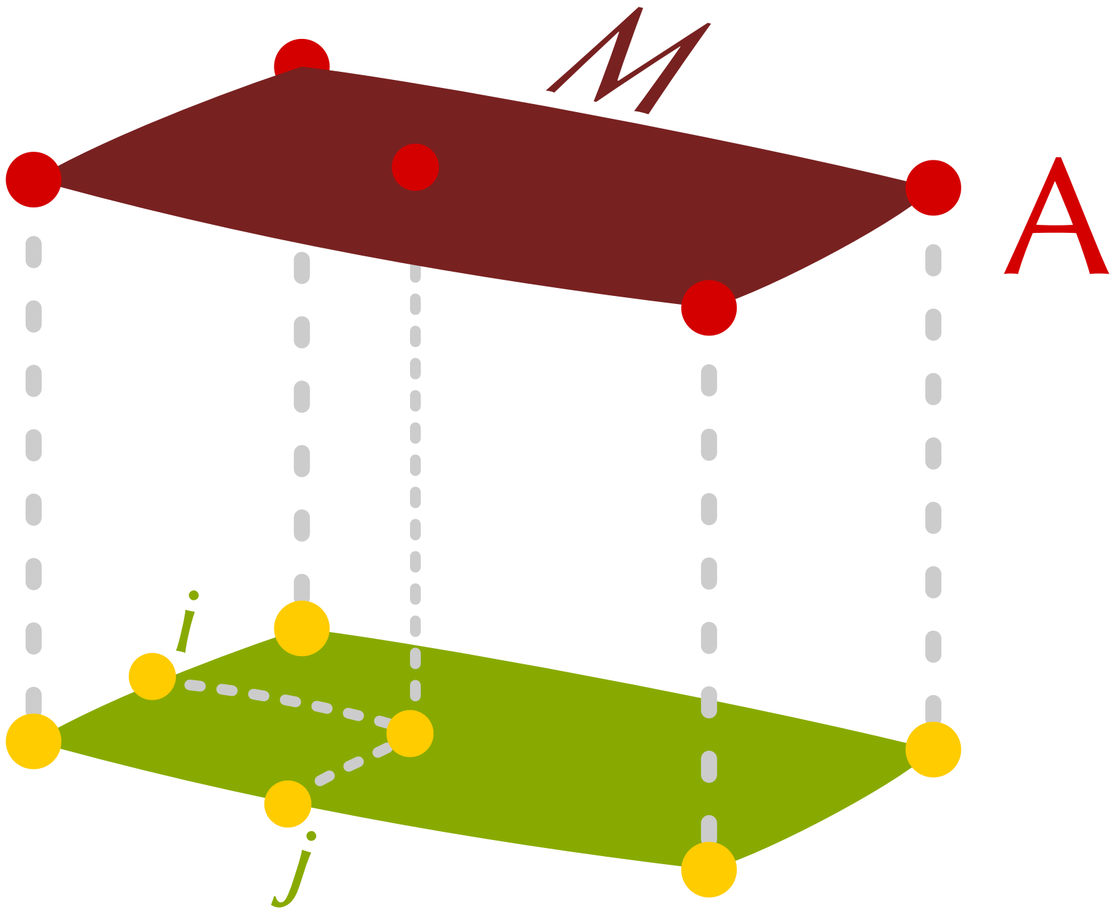
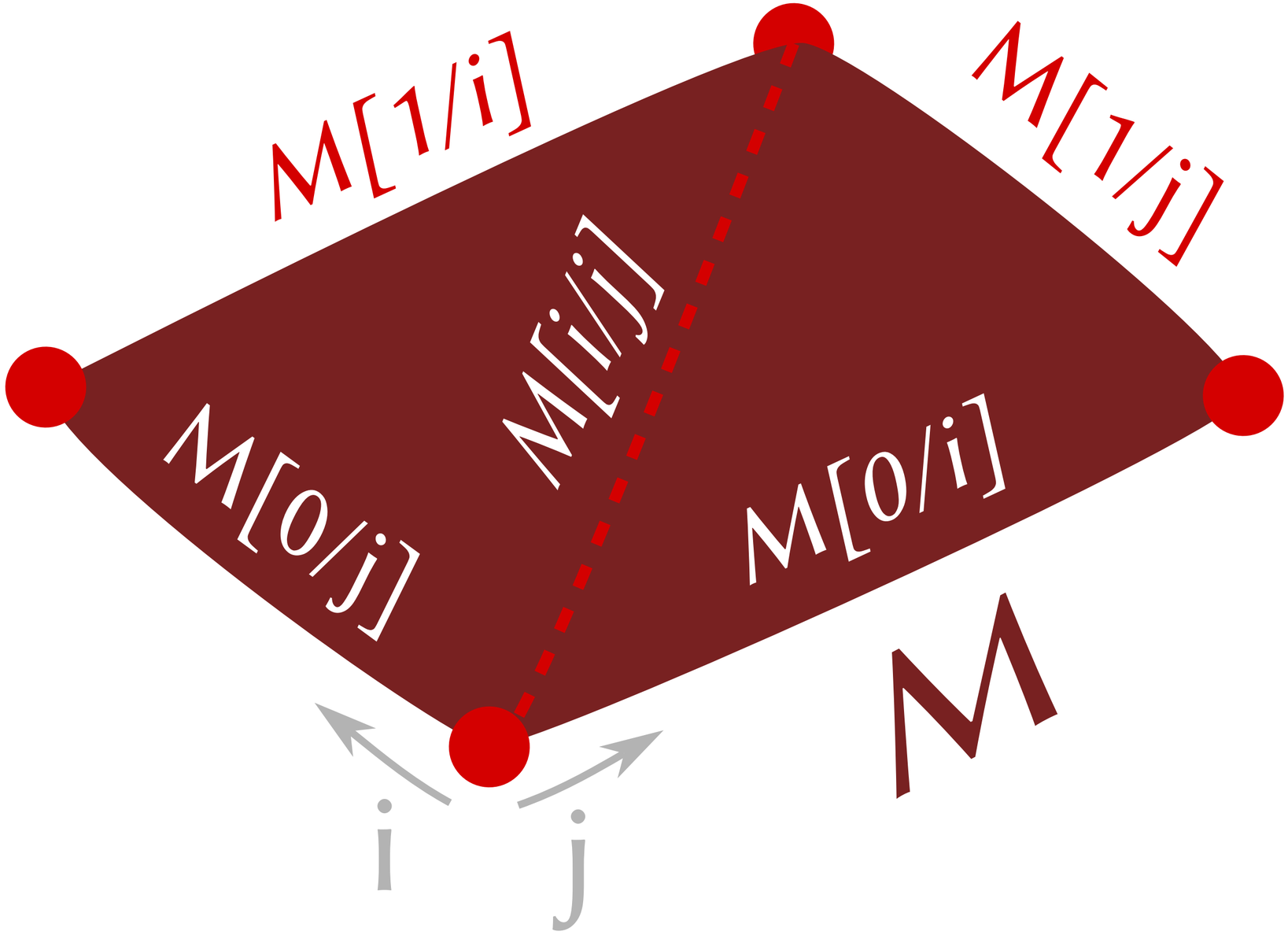


$$i : \mathbb{I} \vdash M : A$$

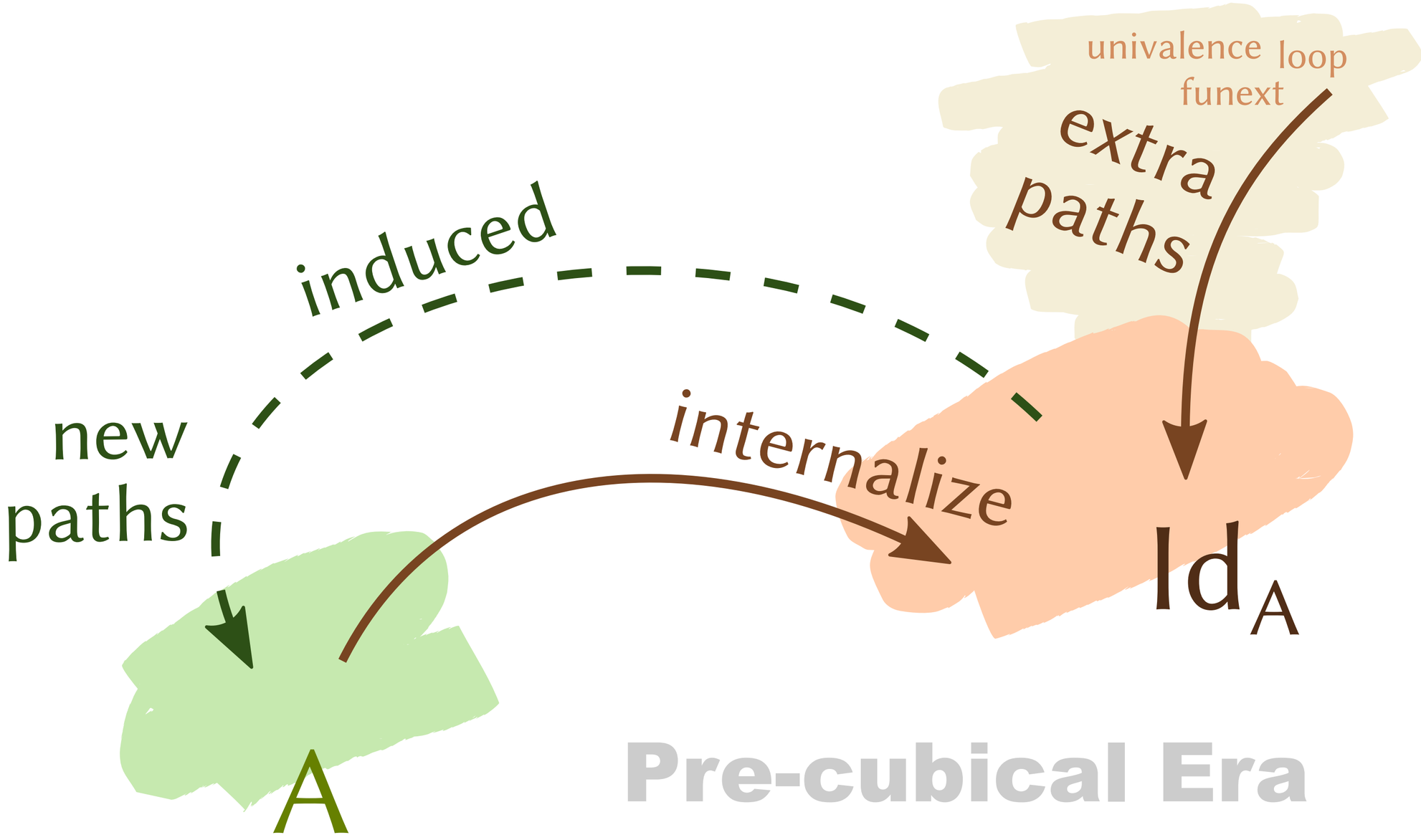


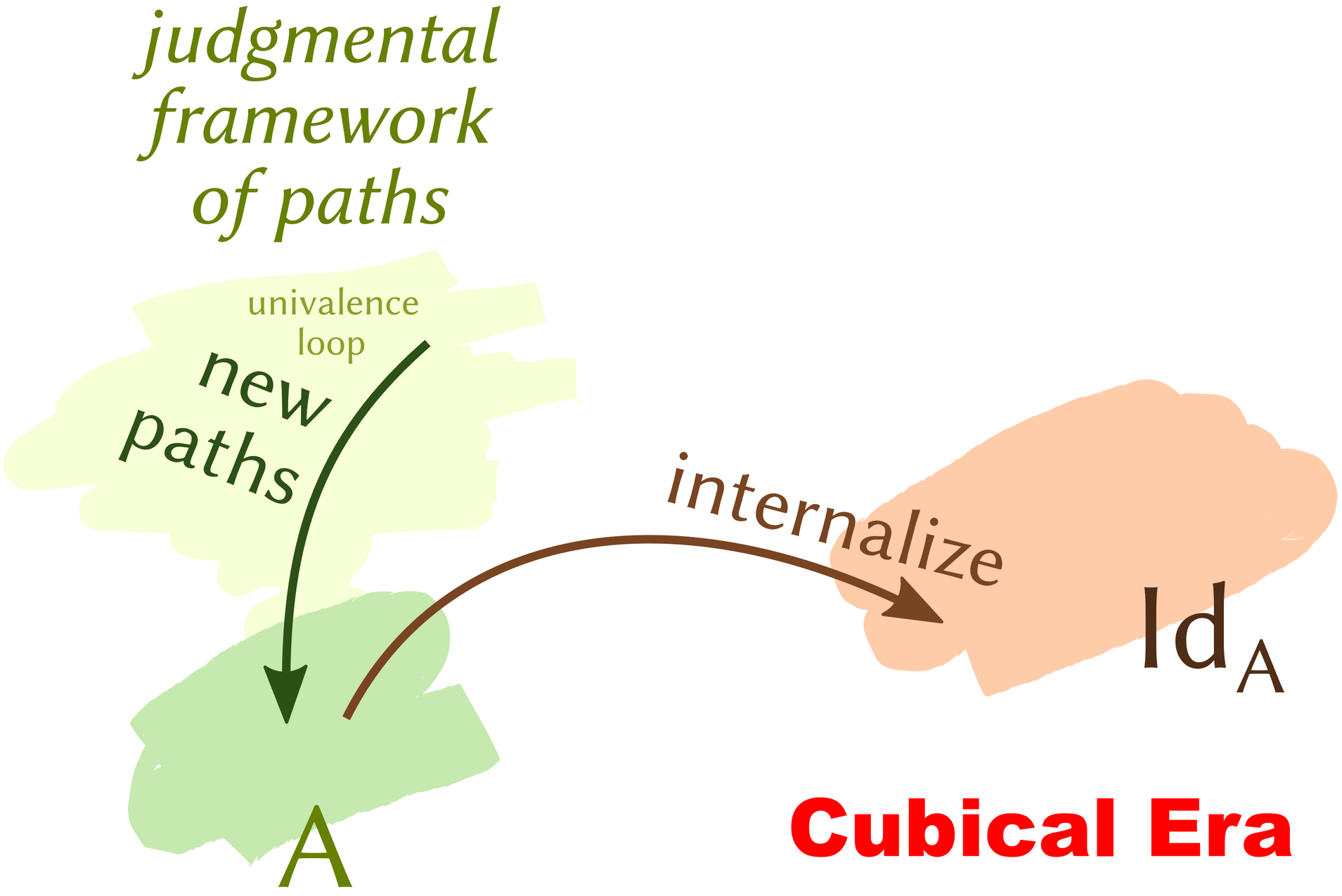
$$i : \mathbb{I},\ j : \mathbb{I} \vdash M : A$$


M[1/i]
M[1/j]
M[i/j]
M[0/j]
M[0/i]
M
i
j

$$\frac{\mathbf{i} : \mathbb{I} \in \Gamma}{\Gamma \vdash \mathbf{i} : \mathbb{I}} \qquad \frac{}{\mathbf{0} : \mathbb{I}} \qquad \frac{}{\mathbf{1} : \mathbb{I}}$$

$$\frac{r : \mathbb{I} \quad s : \mathbb{I}}{r \wedge s : \mathbb{I}} \qquad \frac{r : \mathbb{I} \quad s : \mathbb{I}}{r \vee s : \mathbb{I}} \qquad \frac{r : \mathbb{I}}{\sim r : \mathbb{I}}$$


univalence
loop
funext
extra
paths
induced
new
paths
internalize
Id_A
A
Pre-cubical Era


judgmental framework of paths
univalence loop
new paths
internalize
Id_A
A
Cubical Era

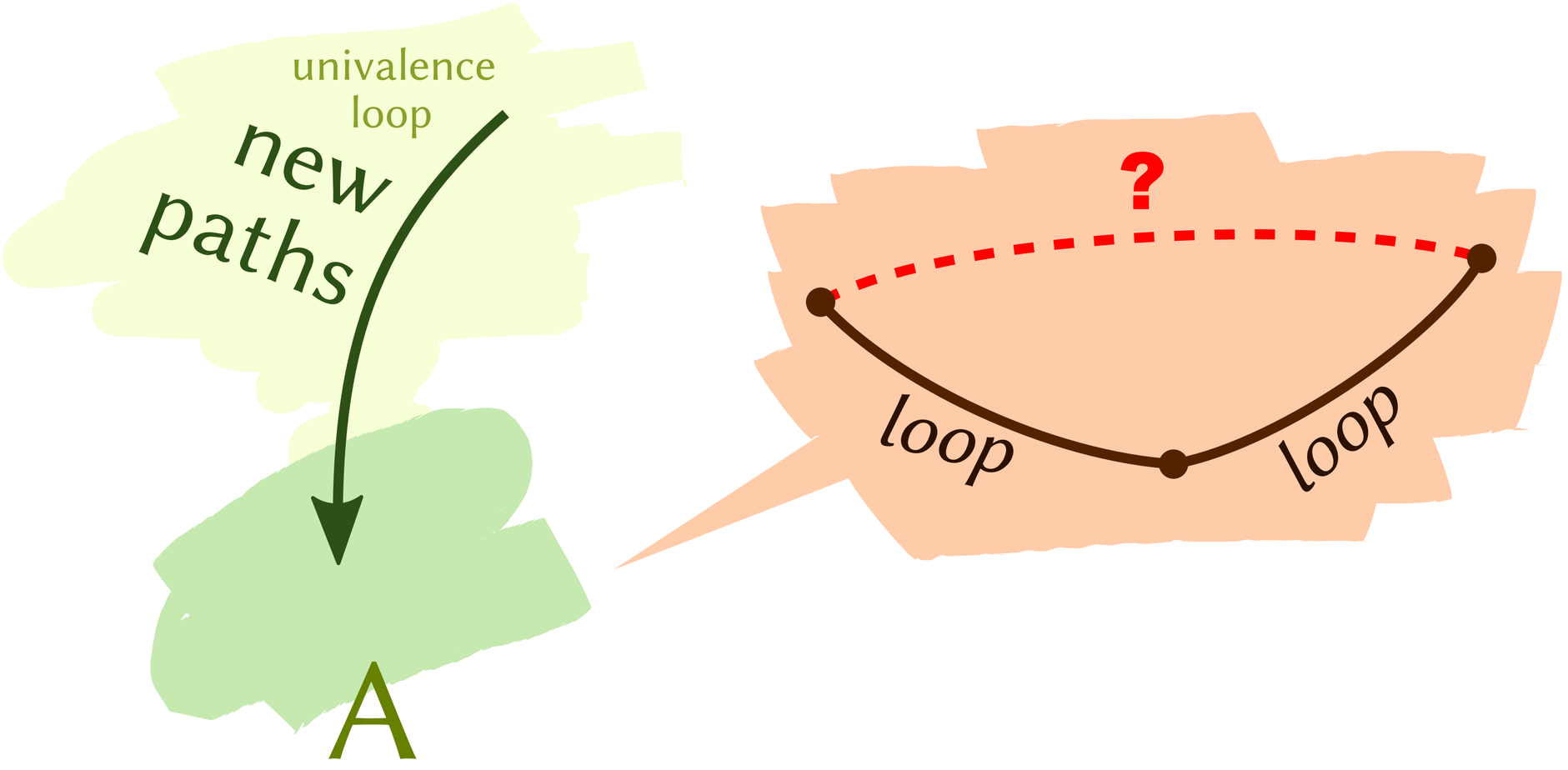
Path types

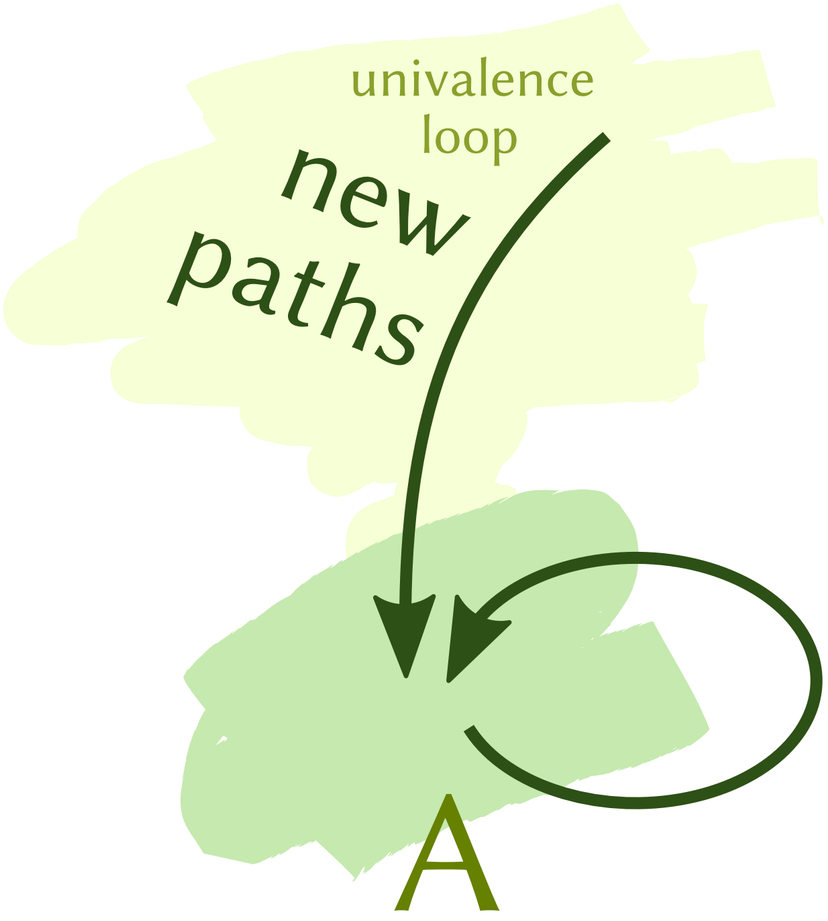
internalized $\mathbf{i}:\mathbb{I} \vdash M : A$

Identification types

freely generated by **refl**

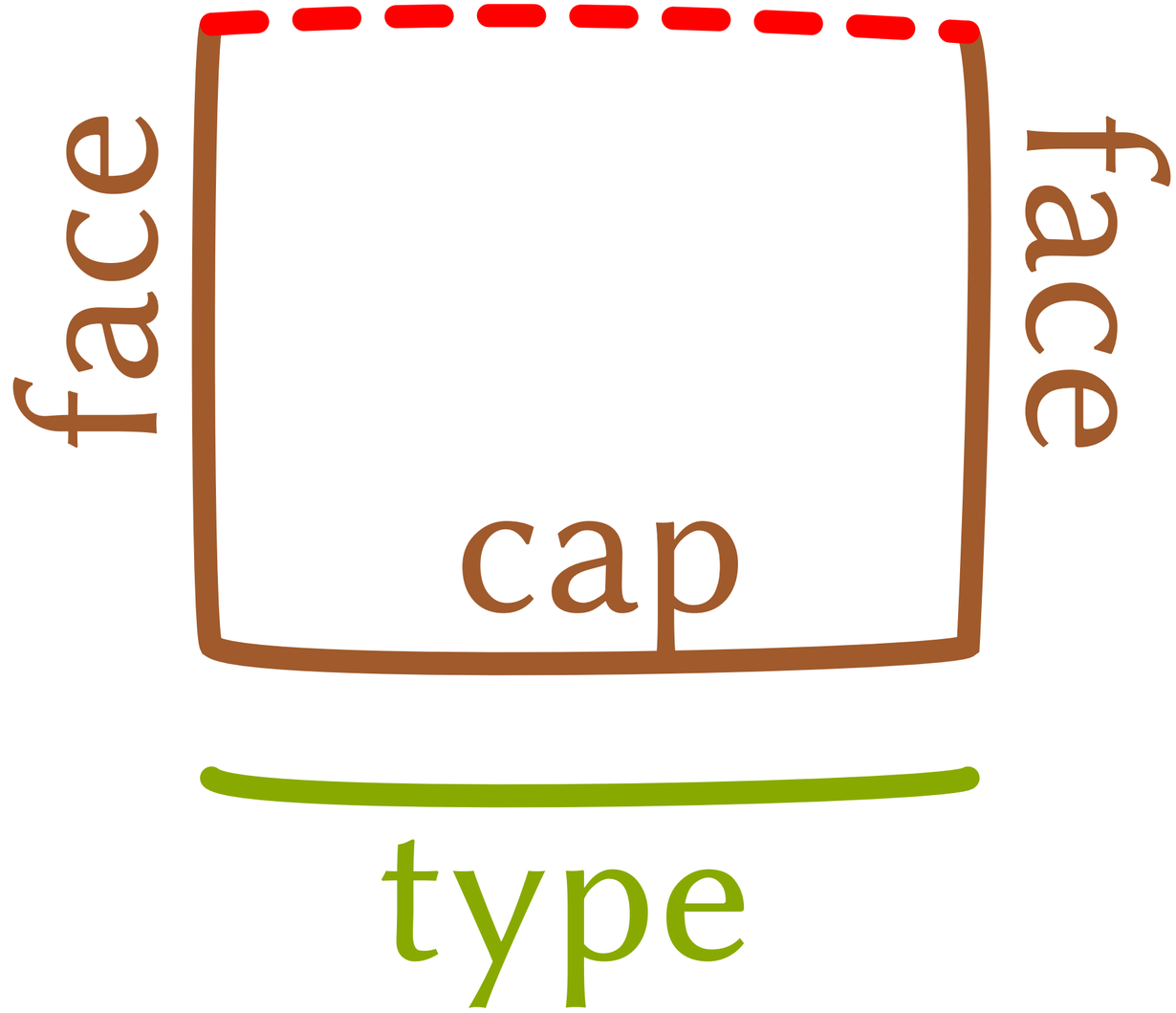
They can co-exist!

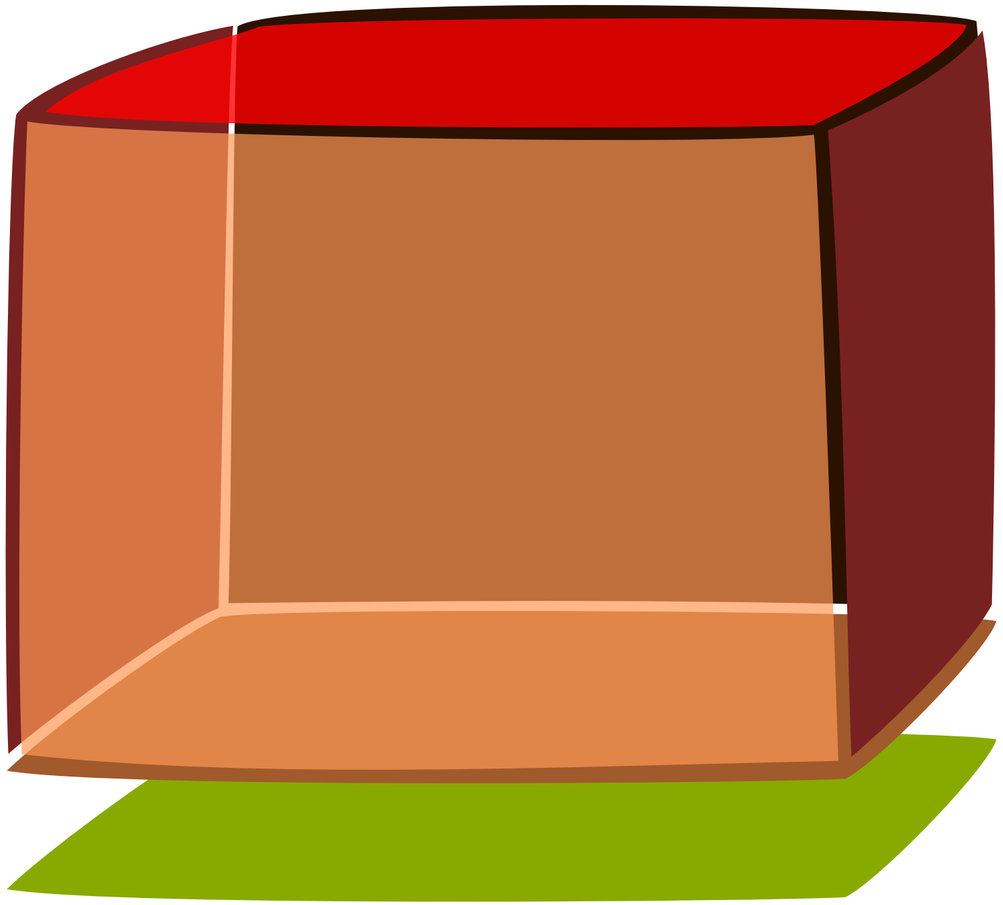

univalence
loop
new
paths
A
?
loop
loop



1. What is the type? (form)
2. What are the constructors? (intro)
3. How to consume an element? (elim)
4. What happens when a constructor is consumed? (β)
5. Elements generated by constructors? (η)
6. How to compose stuff? (Kan operators)

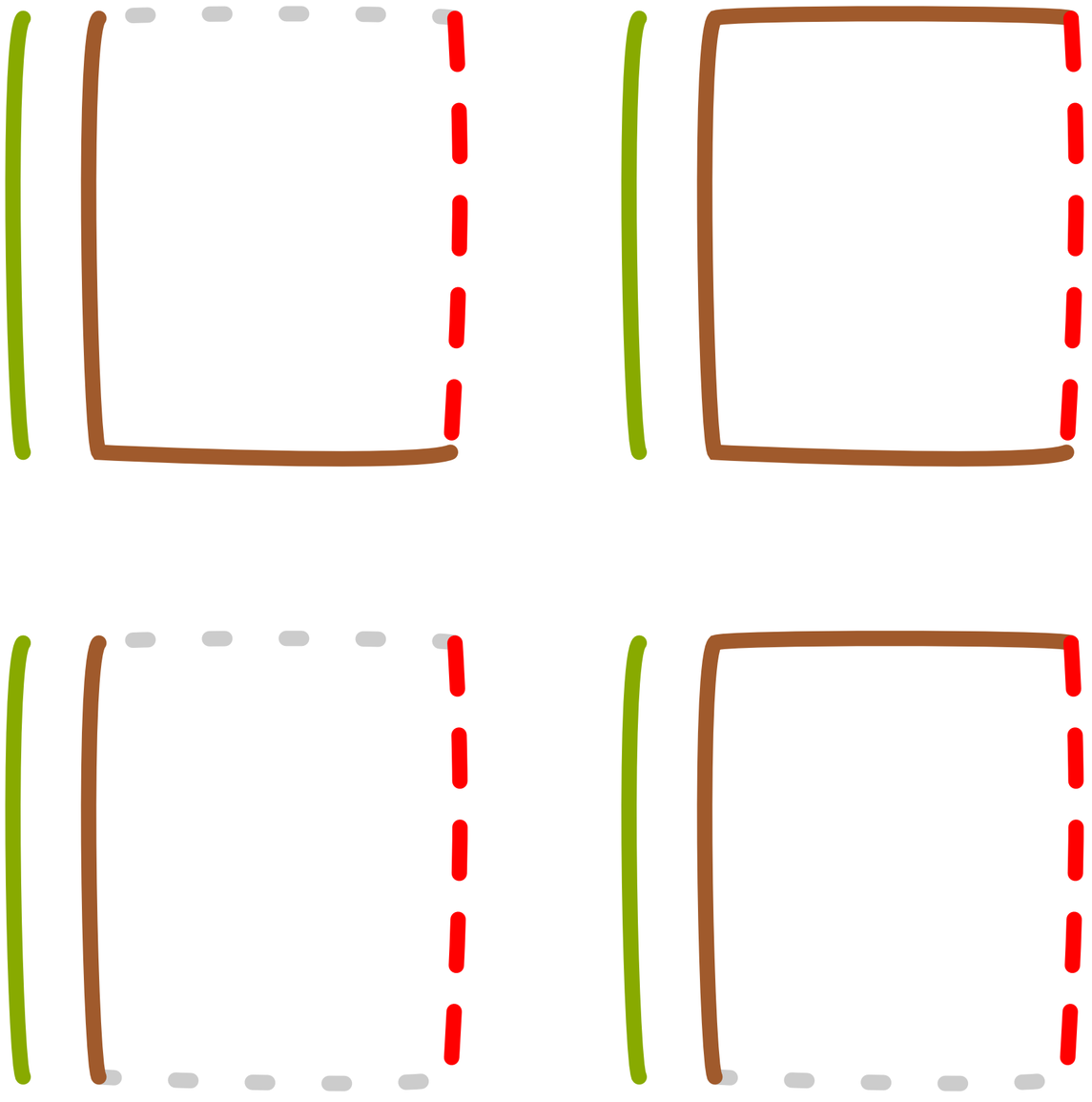
new operators
for every type


face
face
cap
type

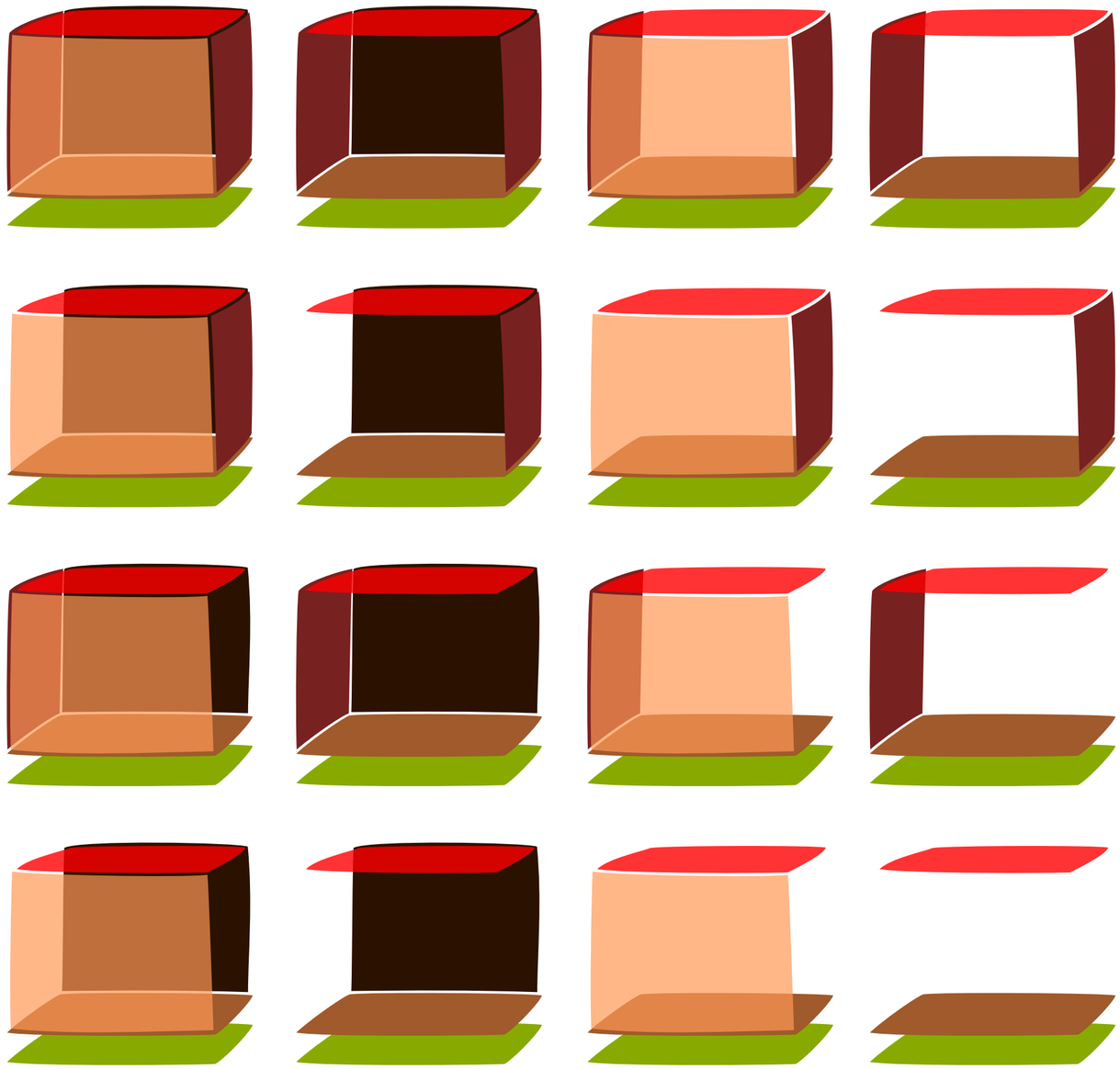


type

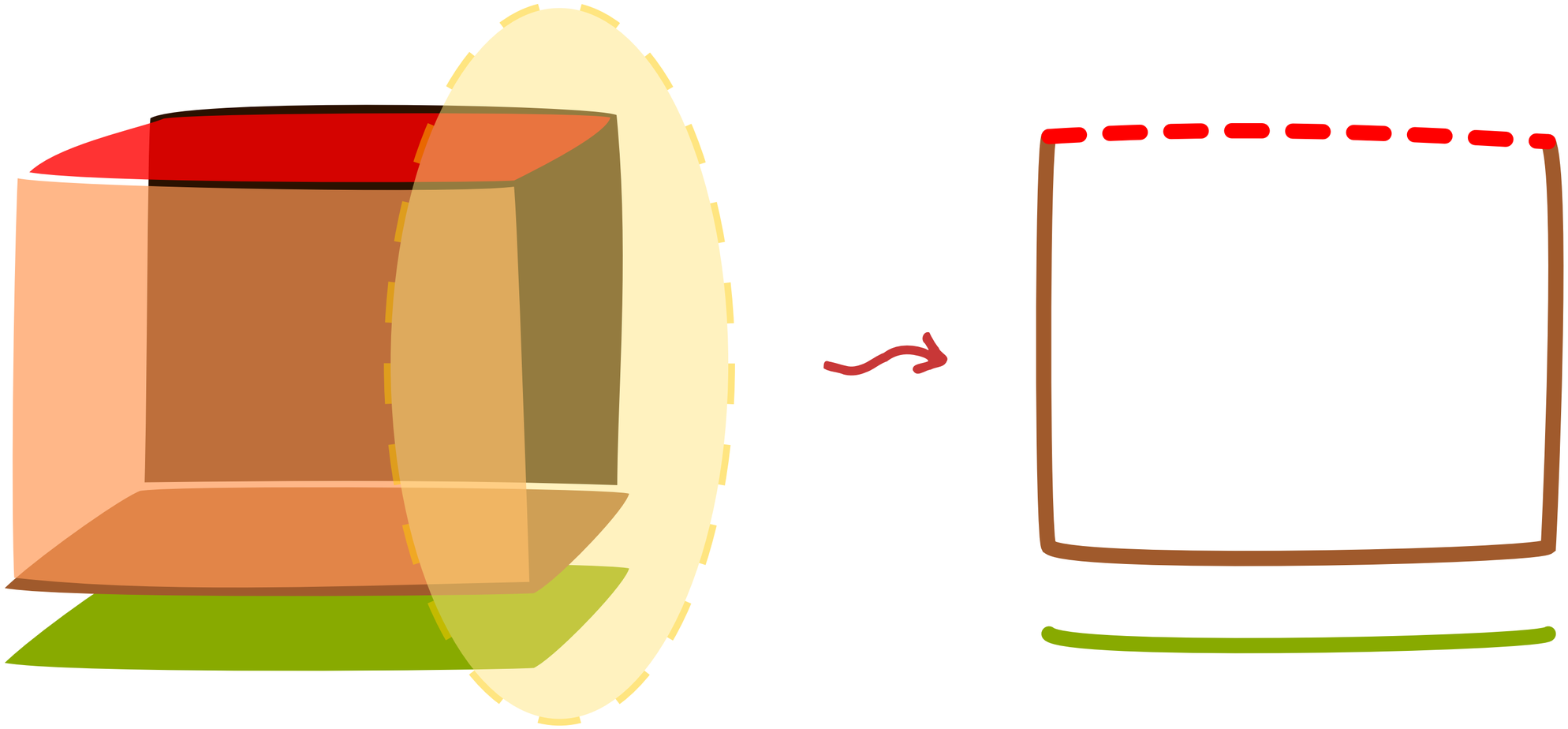
Cubical Agda

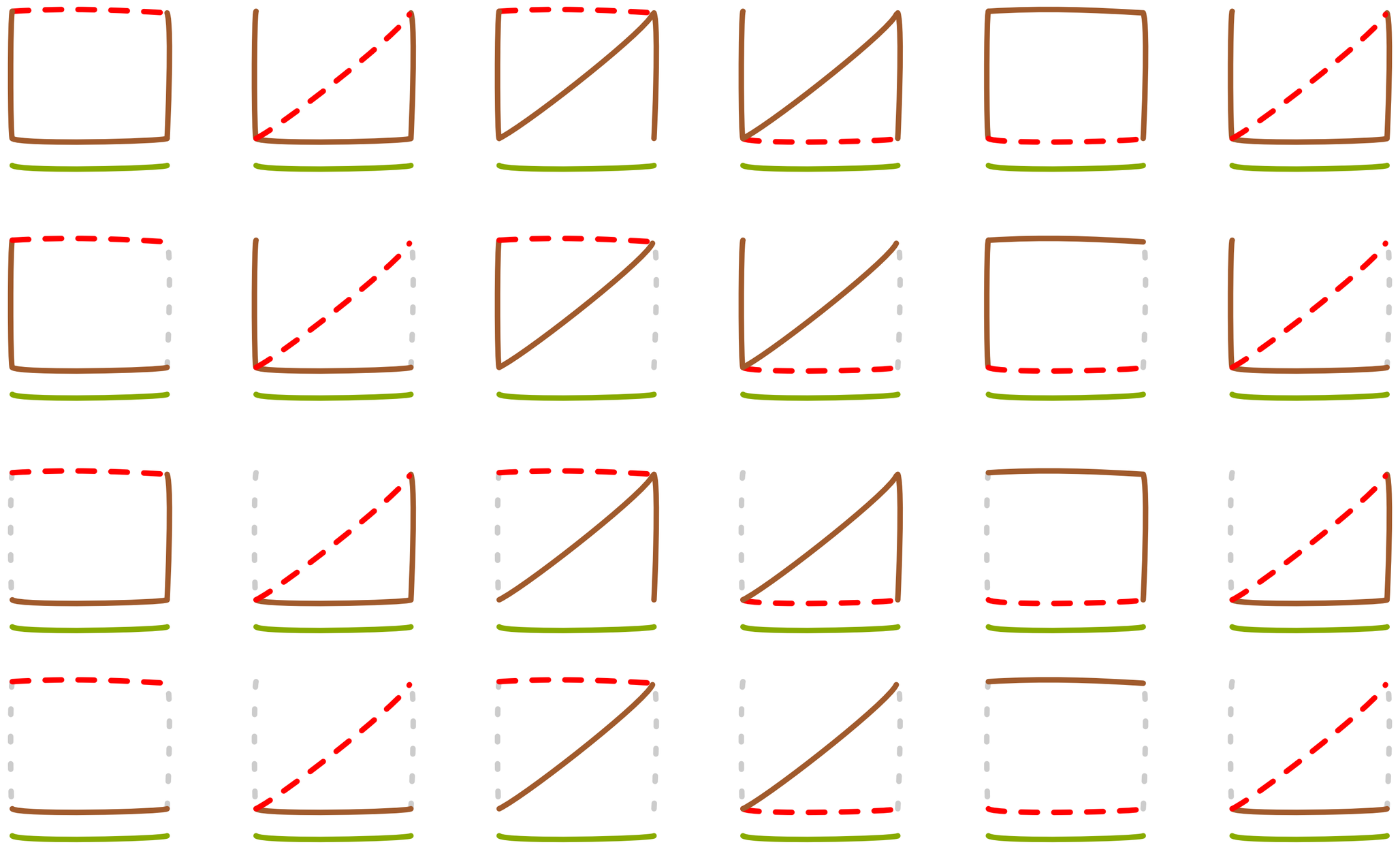


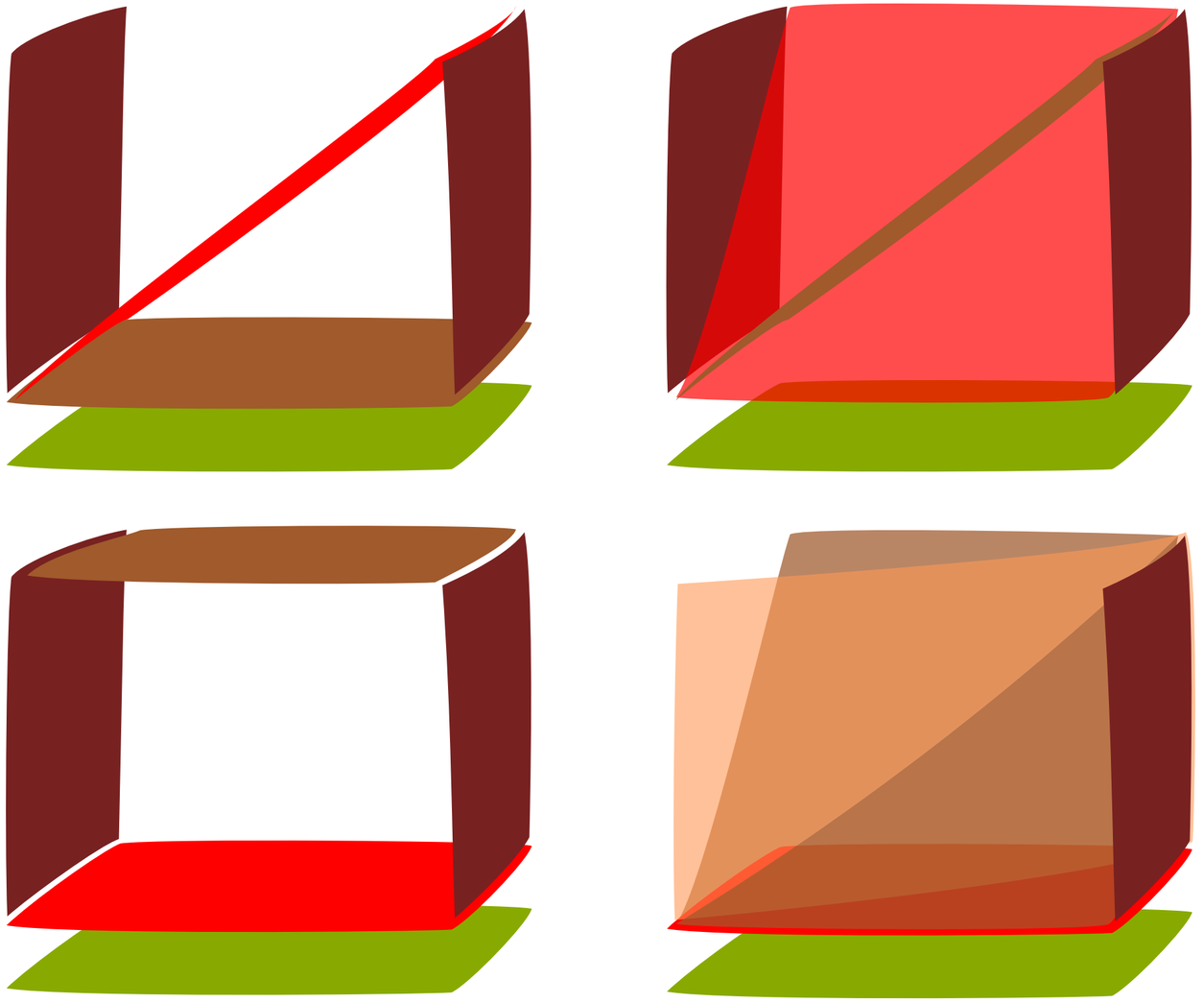
Cubical Agda

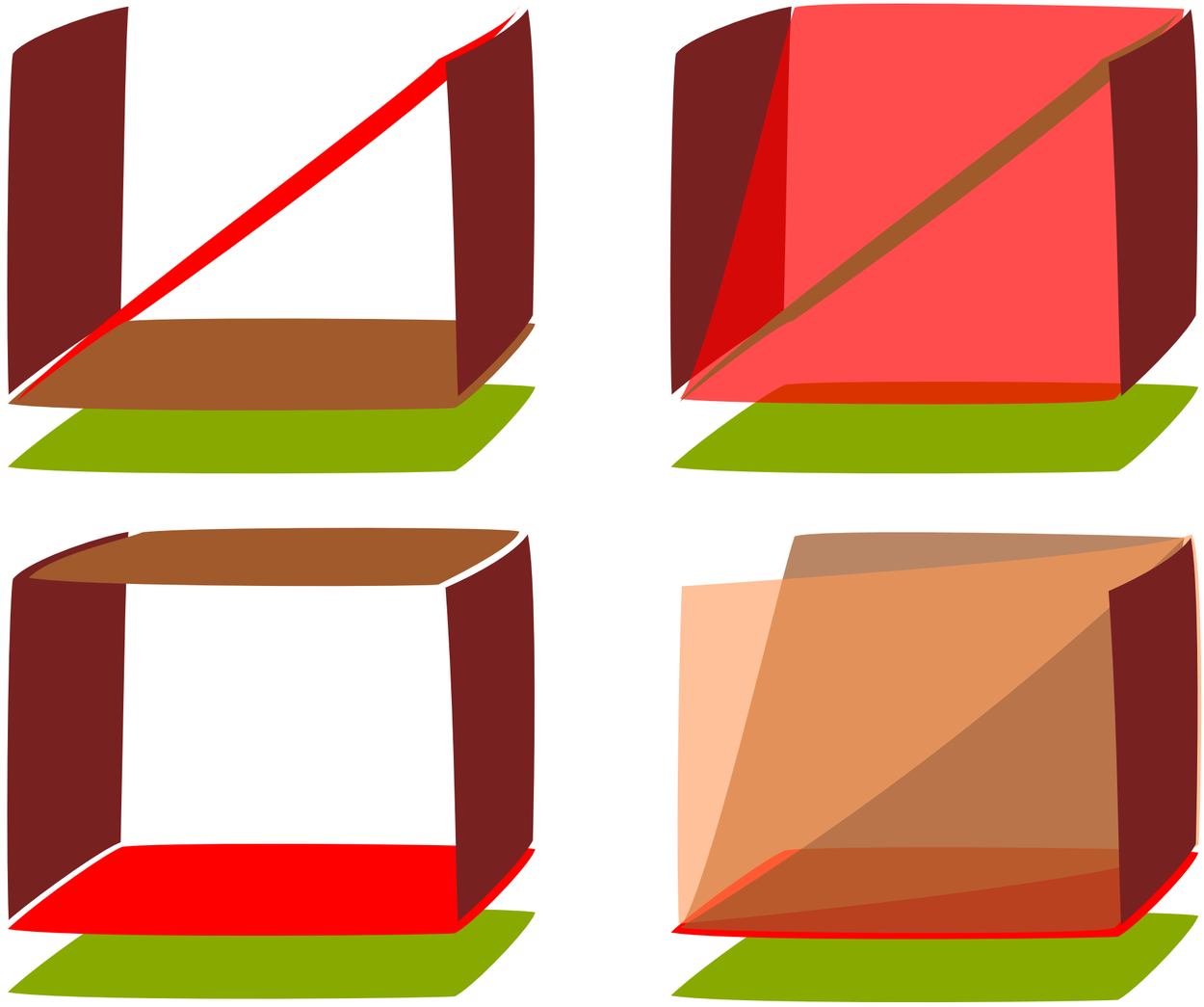


2^{2n-2}



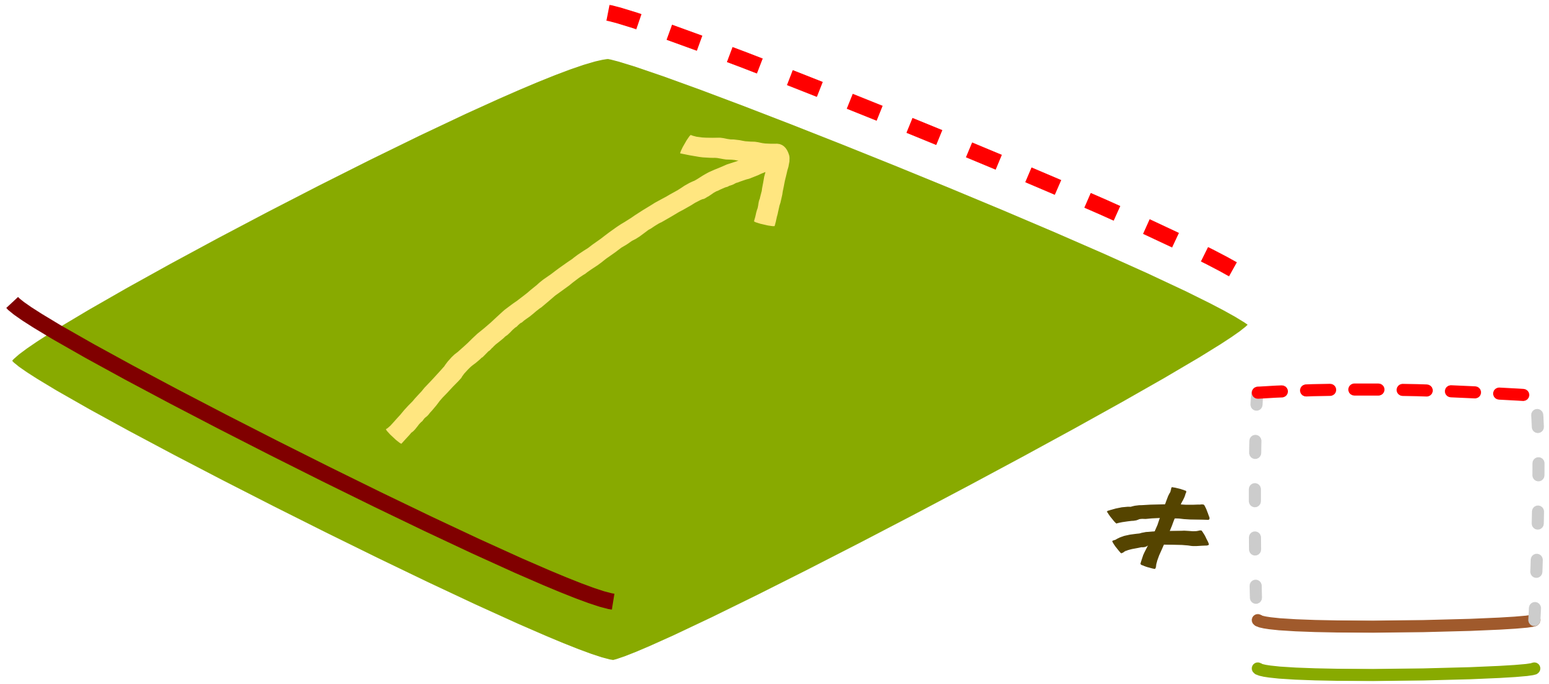


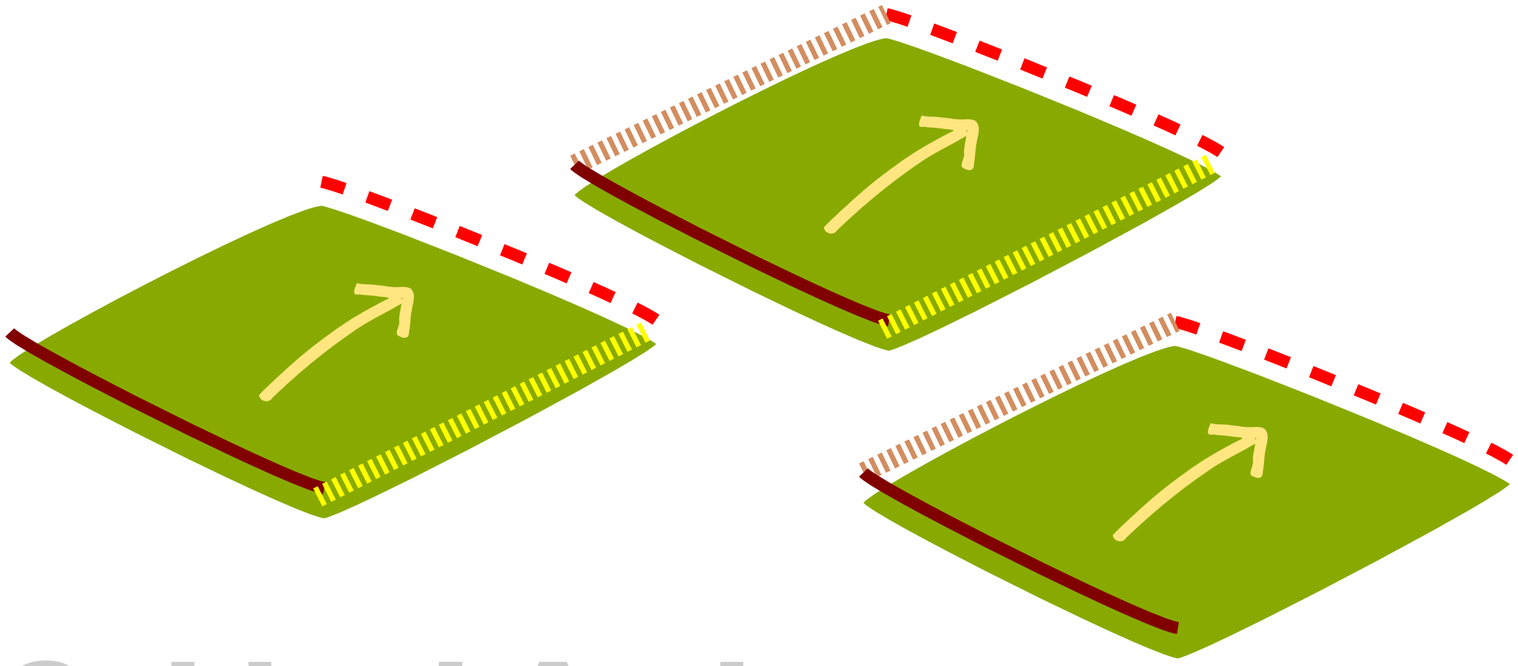




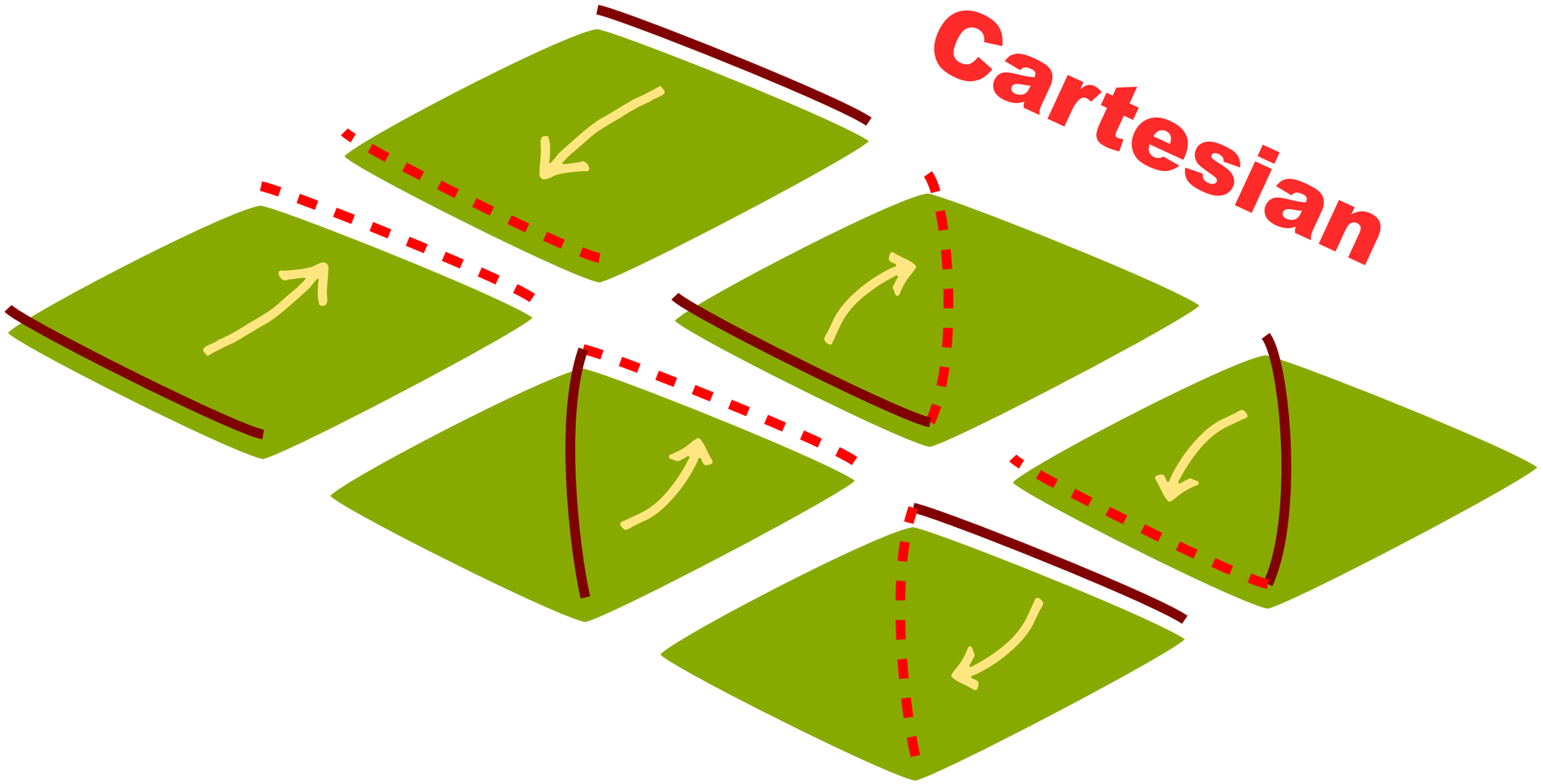
how many

$2^{2n-2} \times (n+1) \times n$?

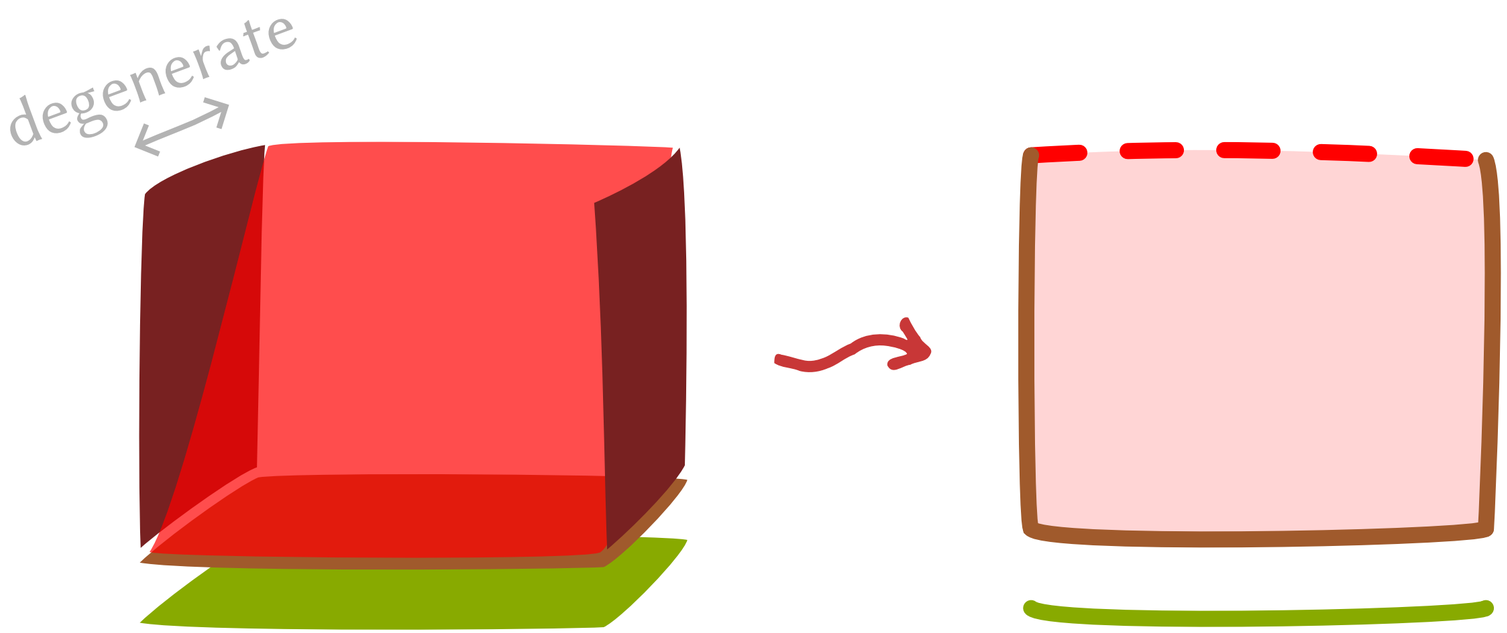


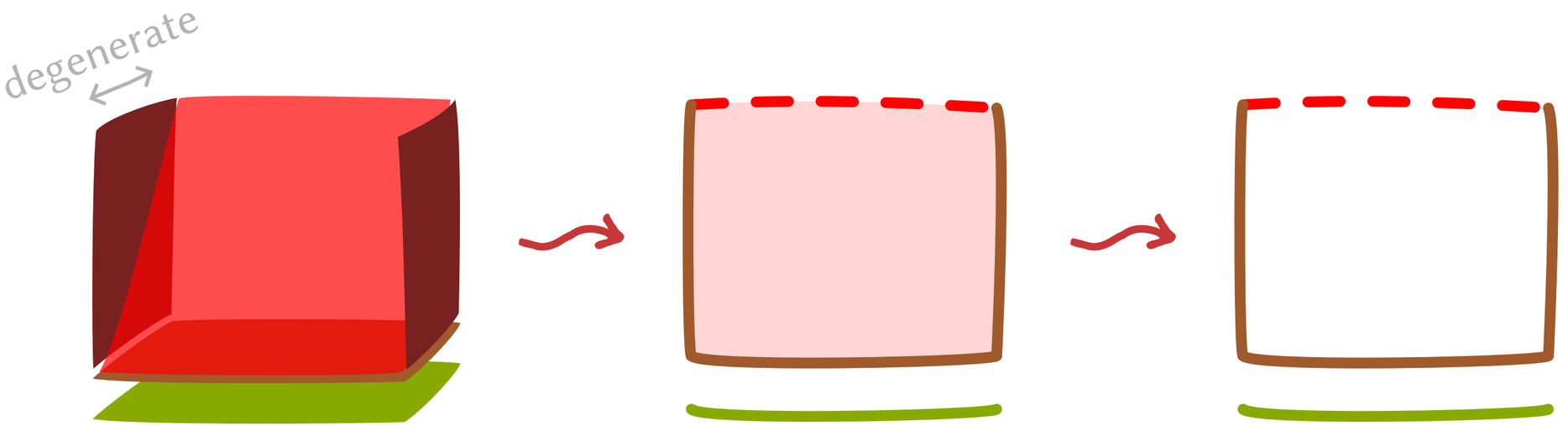


Cubical Agda


Cartesian

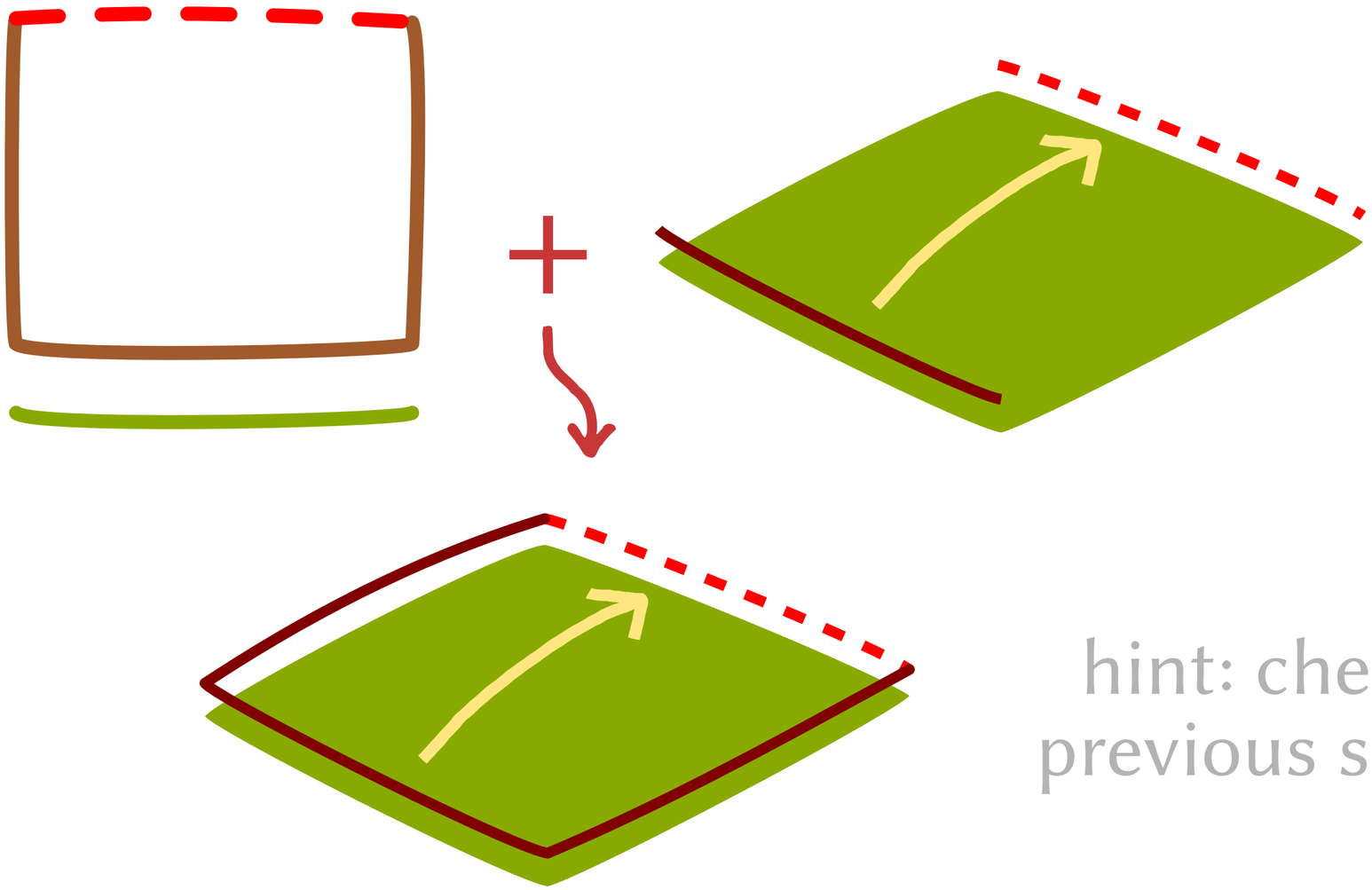
	Cubical Agda [CCHM+CHM]	redtt/cooltt [AFH+ABCHFL+CH]
algebra on $\mathbb{I}$	0, 1, ∧, ∨, ~ De Morgan	0, 1
homogeneous composition	0 to 1, r=0	r to r', r=r'
coercion	with fixation	r to r'


degenerate

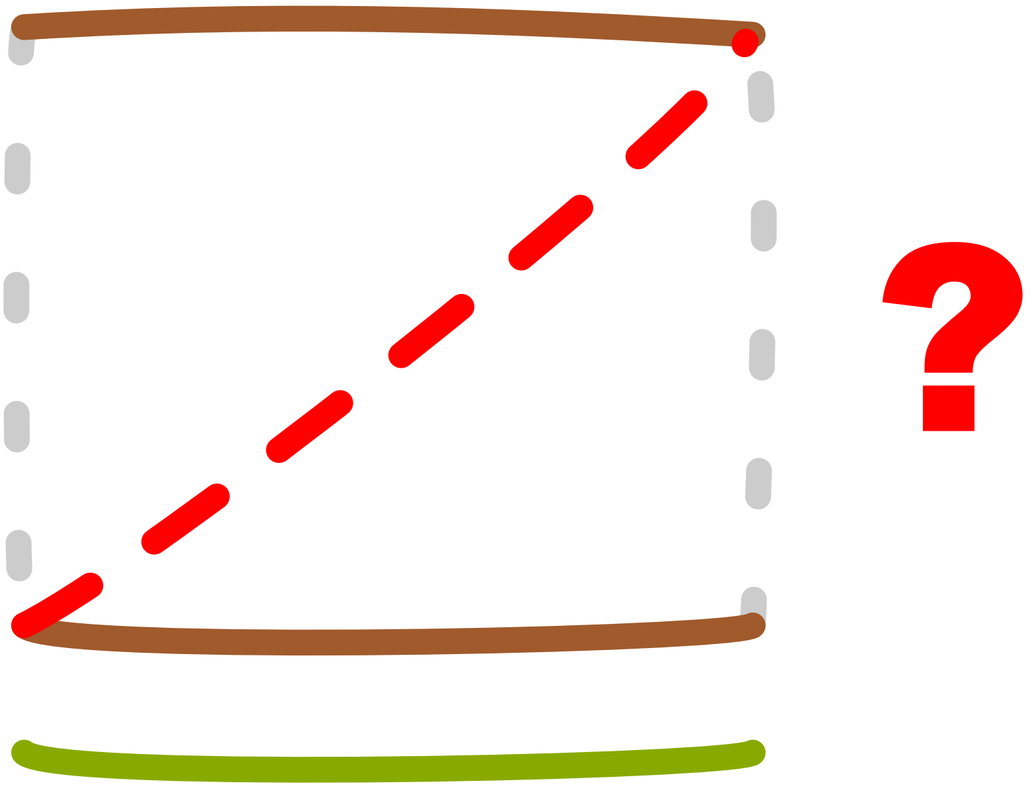


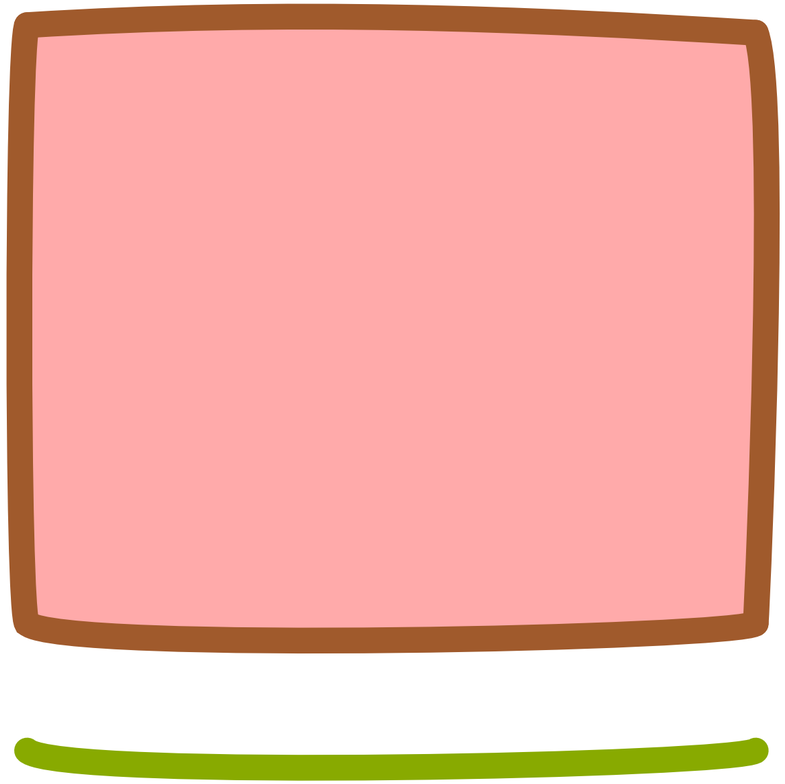
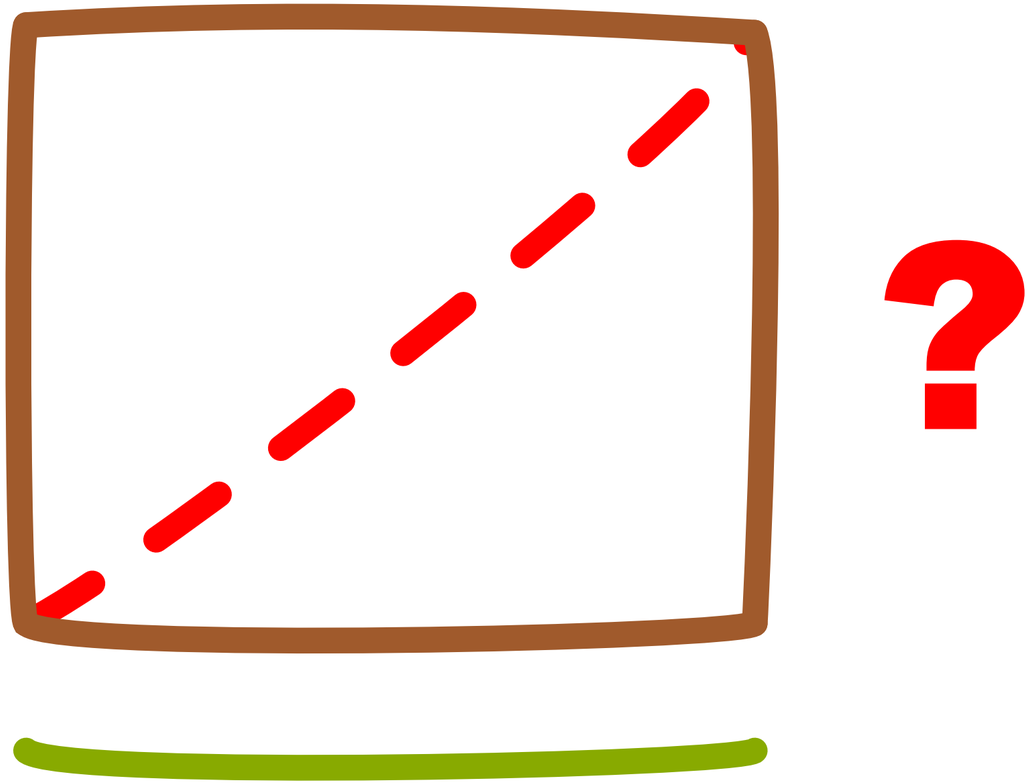
Why Top Lines Agree?

hint: look at the rear



hint: check
previous slide

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