CAT.

THEORY
points in morphisms from
what is \( \bigcirc \)?

any set \( \rightarrow \) unique morphism \( \rightarrow \) terminal
points in points in
morphism from morphisms from terminal

points in
morphism from

terminal
Abstraction

category theory

<--

type theory
Universal Properties

\(\mathbb{N}\)  most general \(\mathbb{N}\)-algebra

\(S1\)  most general \(S1\)-algebra

\(\text{Id}(M; N)\)  freely generated by \text{refl}

n-truncation  best n-type approximation
Categorical Aspects

1. connectives in type theory

\[ \Pi, \Sigma, \top, \bot, \text{Path} \]

universal properties in some category
2. all objects that look like a type theory
Comprehension categories
Categories with families
Categories with attributes
Display map categories
Path
\[ \Sigma \Pi \top \bot \]
interpretations as morphisms
A type theory is the most general object that looks like a type theory.
models of the type theory

(the most general model is the theory itself)
Normalization and other meta-theorems follow from the syntax being the most general.
Further Readings

Categorical Logic and Type Theory by Bart Jacobs

Categorical Logic by Andrew Pitts