U

Coercion

Composition

Univalence

# Coercion

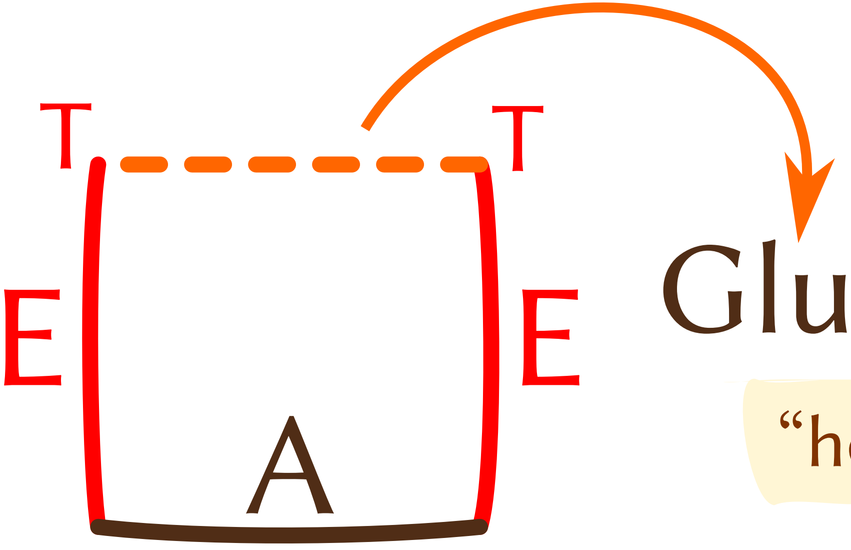

$$\text{transp}^{i}\ U\ \varphi\ A \equiv A$$

*(as in inductive types)*

Composition

Univalence

→ Glue

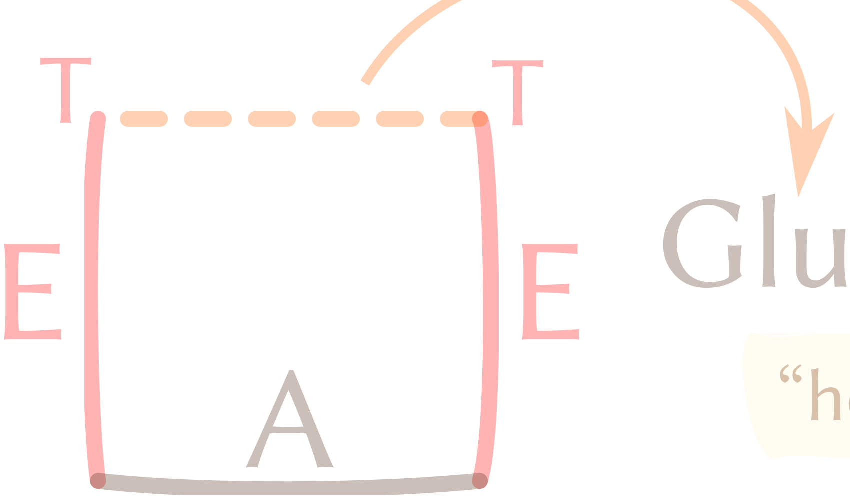



Glue $[\varphi \mapsto (T, E)]\ A$

“$\text{hcomp}^i\ U\ [\varphi \mapsto ua(E)@i]\ A$”

$\varphi \vdash E : T \simeq A$

(an equivalence)




$\mathrm{Glue}\ [\varphi \mapsto (T, E)]\ A$

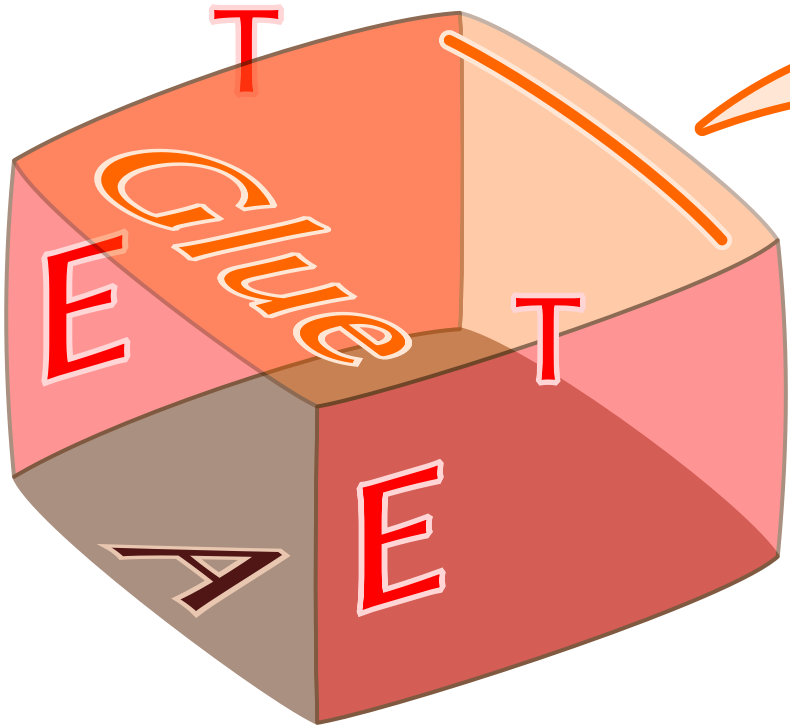
"$\mathrm{hcomp}^i\ U\ [\varphi \mapsto \mathrm{ua}(E)@i]\ A$"

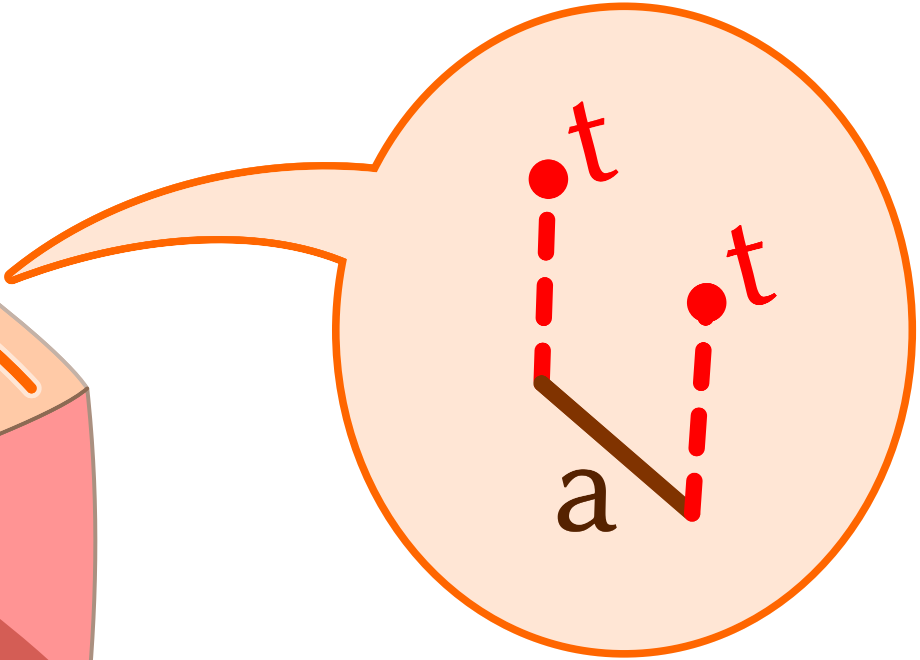
$\varphi \vdash E : T \simeq A$

(an equivalence)

# Glue is a type!

## It has hcomp & transp



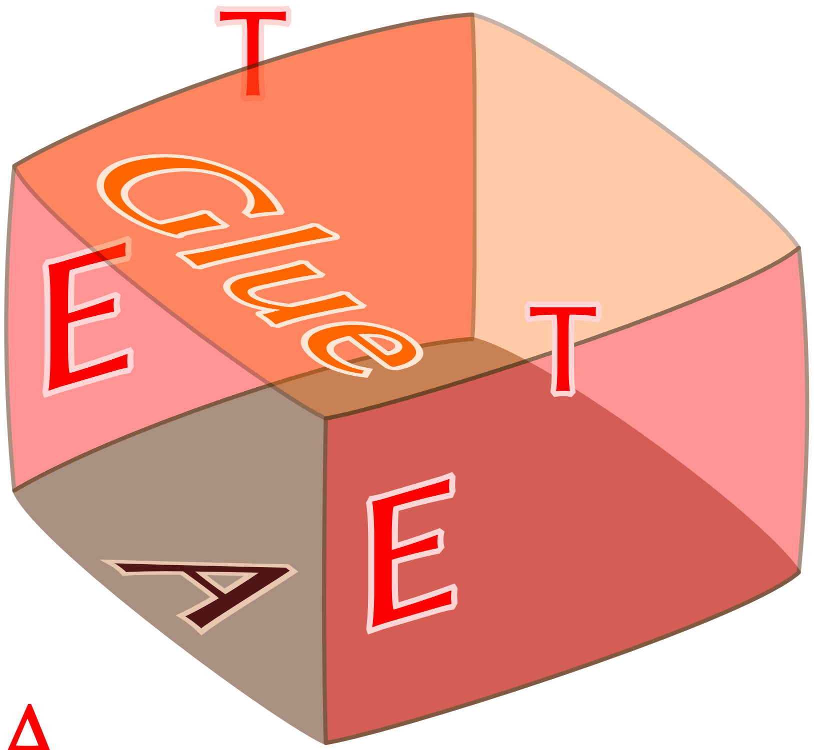




$$\mathsf{glue}\ [\varphi \mapsto t]\ a$$

$$\mathsf{unglue}\ u : A$$

$$\mathsf{unglue}\ (\mathsf{glue}\ [\varphi \mapsto t]\ a) \equiv a$$

$$\mathsf{Glue}\ [\varphi \mapsto (T, E)]\ A$$




$$A : U$$

$$\varphi \vdash T : U$$

$$\varphi \vdash E : T \simeq A$$

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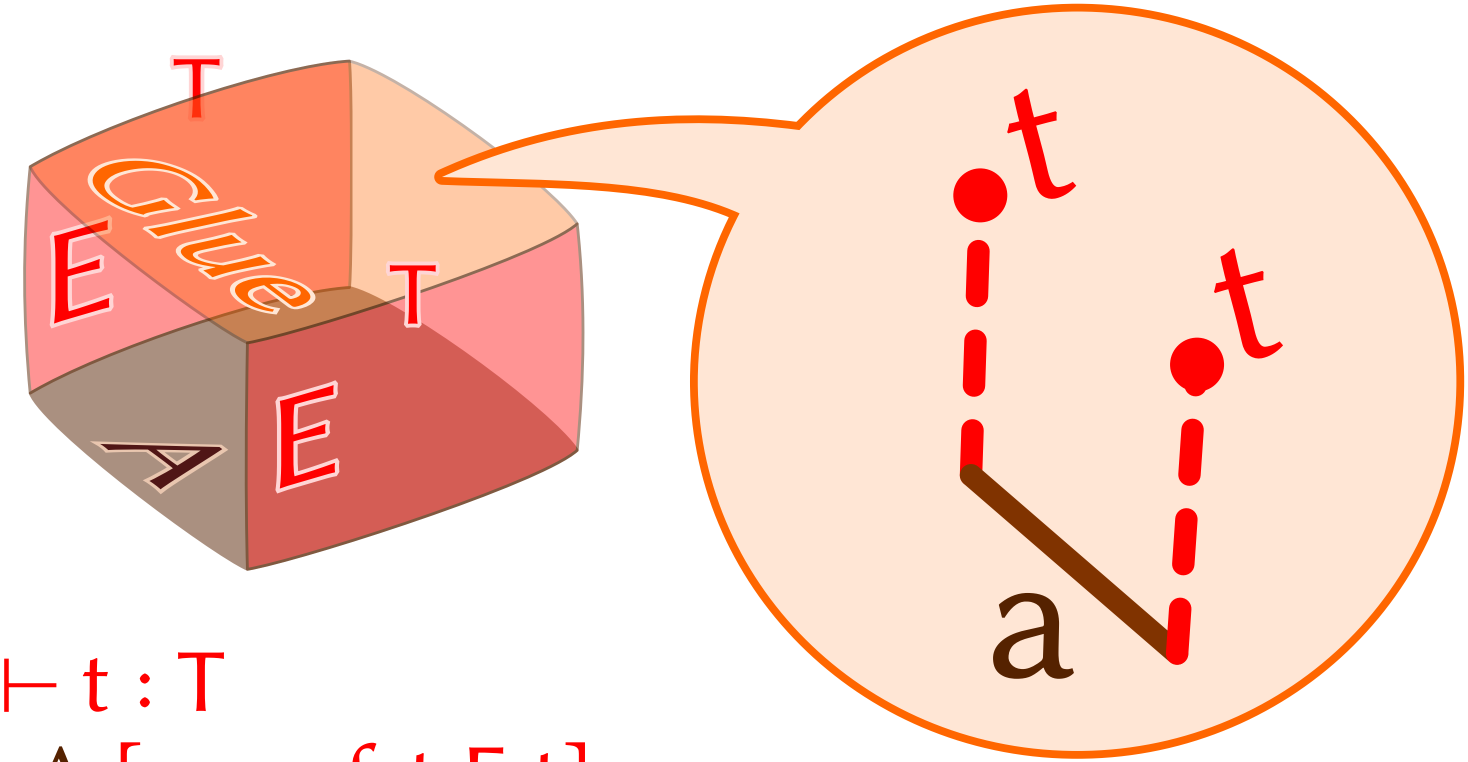
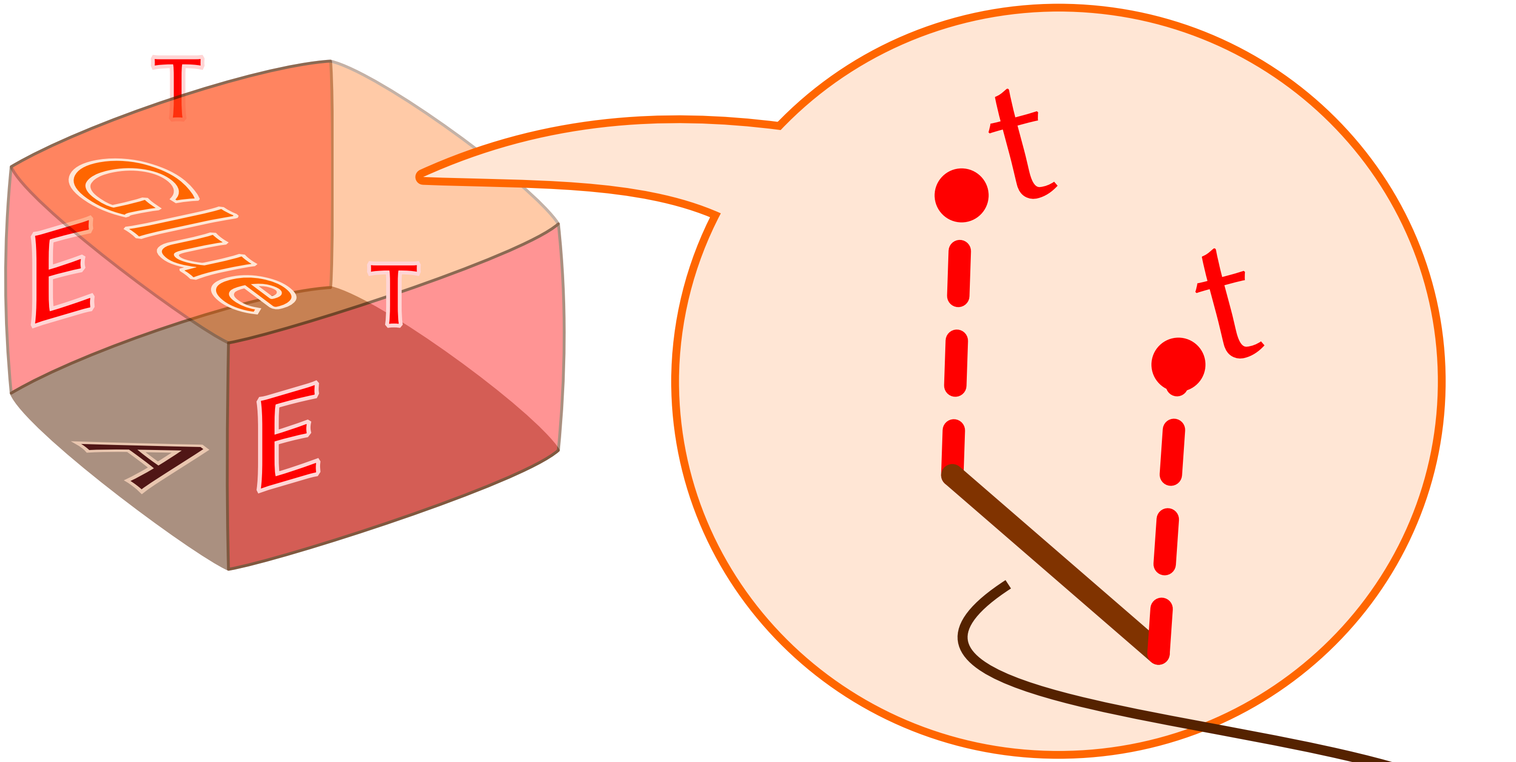
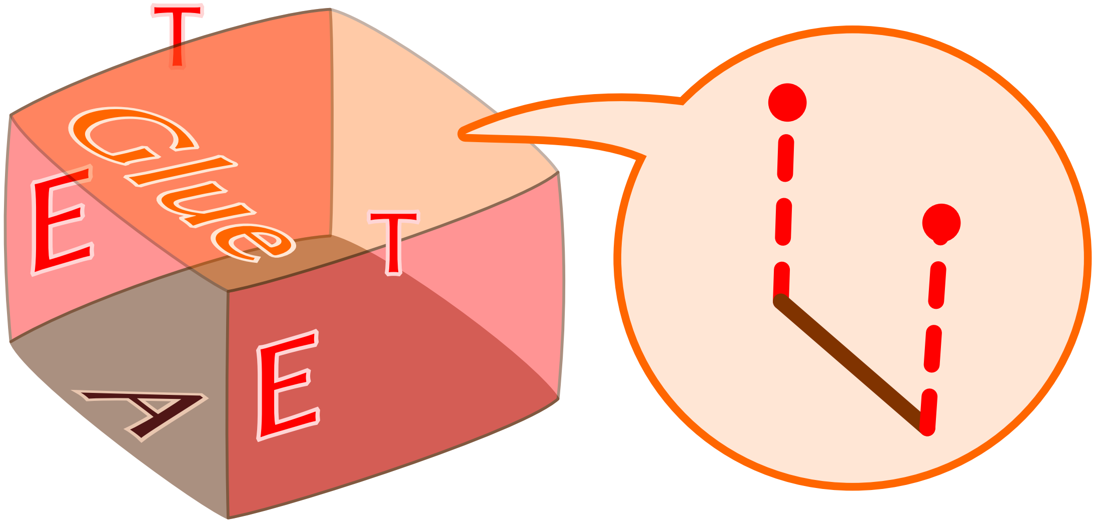
$$\text{Glue}\ [\varphi \mapsto (T, E)]\ A : U\ [\varphi \mapsto T]$$




$$\frac{\varphi \vdash t : T \qquad a : A\ [\varphi \mapsto \mathrm{fst}\ E\ t]}{\mathrm{glue}\ [\varphi \mapsto t]\ a : (\mathrm{Glue}\ [\varphi \mapsto (T, E)]\ A)\ [\varphi \mapsto t]}$$




$$\frac{u : \mathsf{Glue}\ [\varphi \mapsto (T, E)]\ A}{\mathsf{unglue}\ [\varphi \mapsto E]\ u : A\ [\varphi \mapsto \mathsf{fst}\ E\ u]}$$



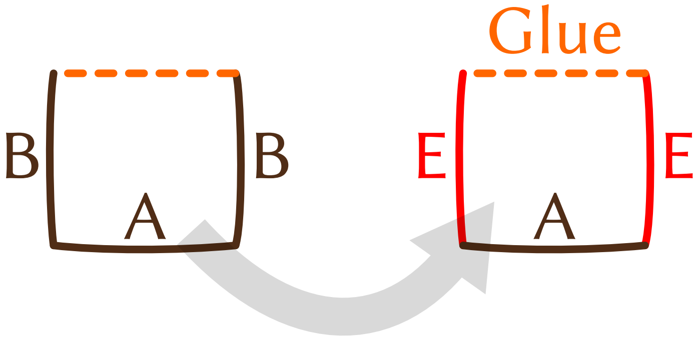

$$\mathsf{unglue}\ [\varphi \mapsto E]\ (\mathsf{glue}\ [\varphi \mapsto t]\ a) \equiv a : A$$

$$u \equiv \mathsf{glue}\ [\varphi \mapsto u]\ (\mathsf{unglue}\ [\varphi \mapsto E]\ u)$$

$$: \mathsf{Glue}\ [\varphi \mapsto (T, E)]\ A$$

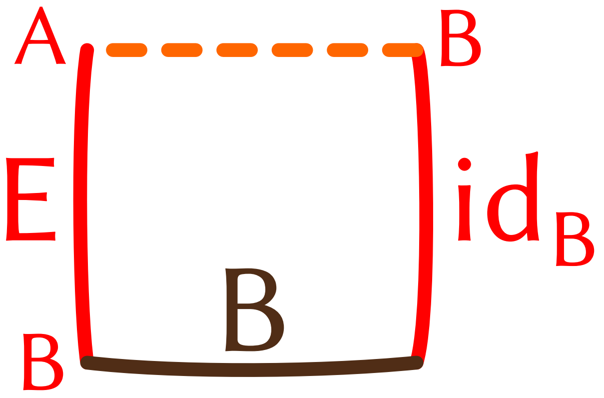
Composition
Univalence
→ Glue

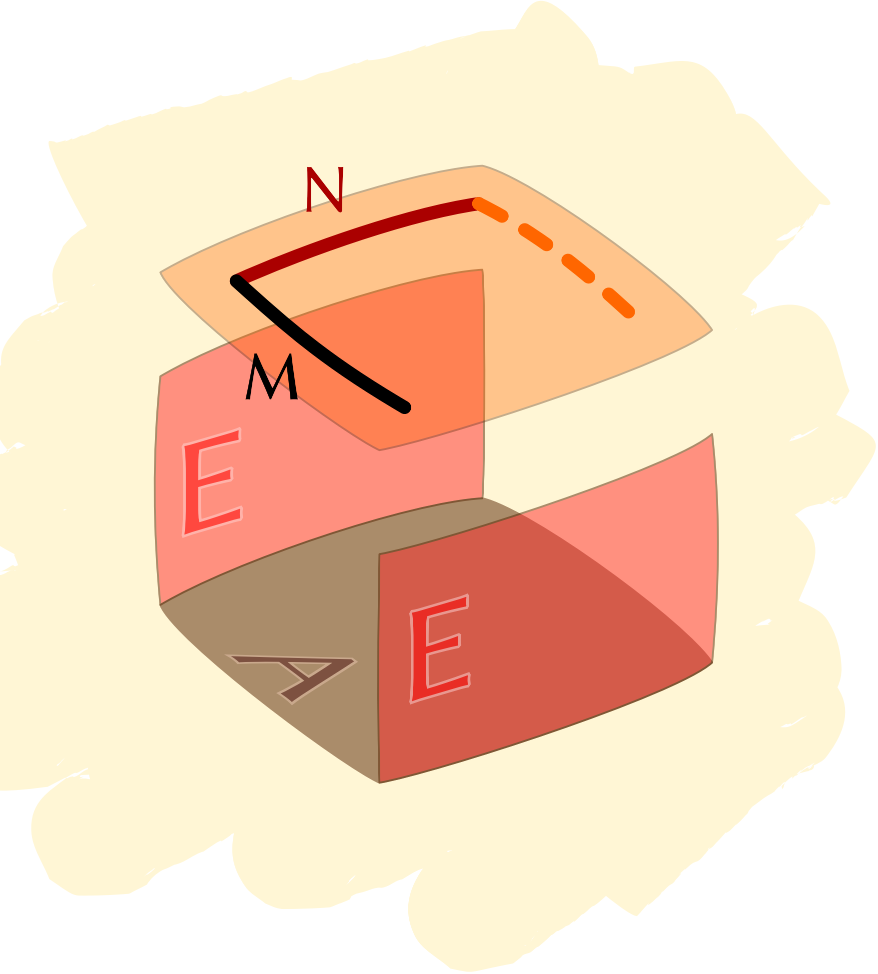
## Composition



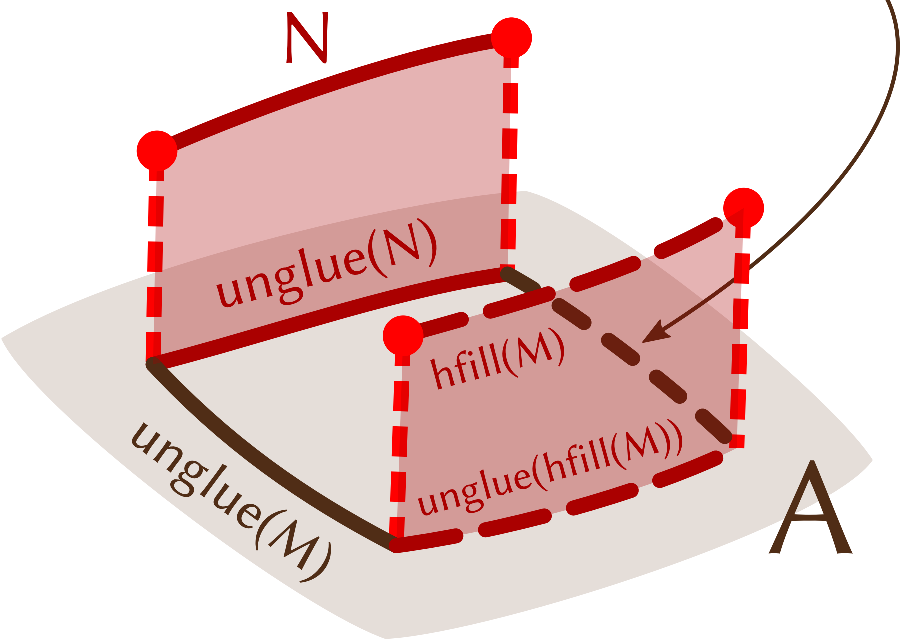

E :≡ transp$^{i}$ (B@(~i)) 0

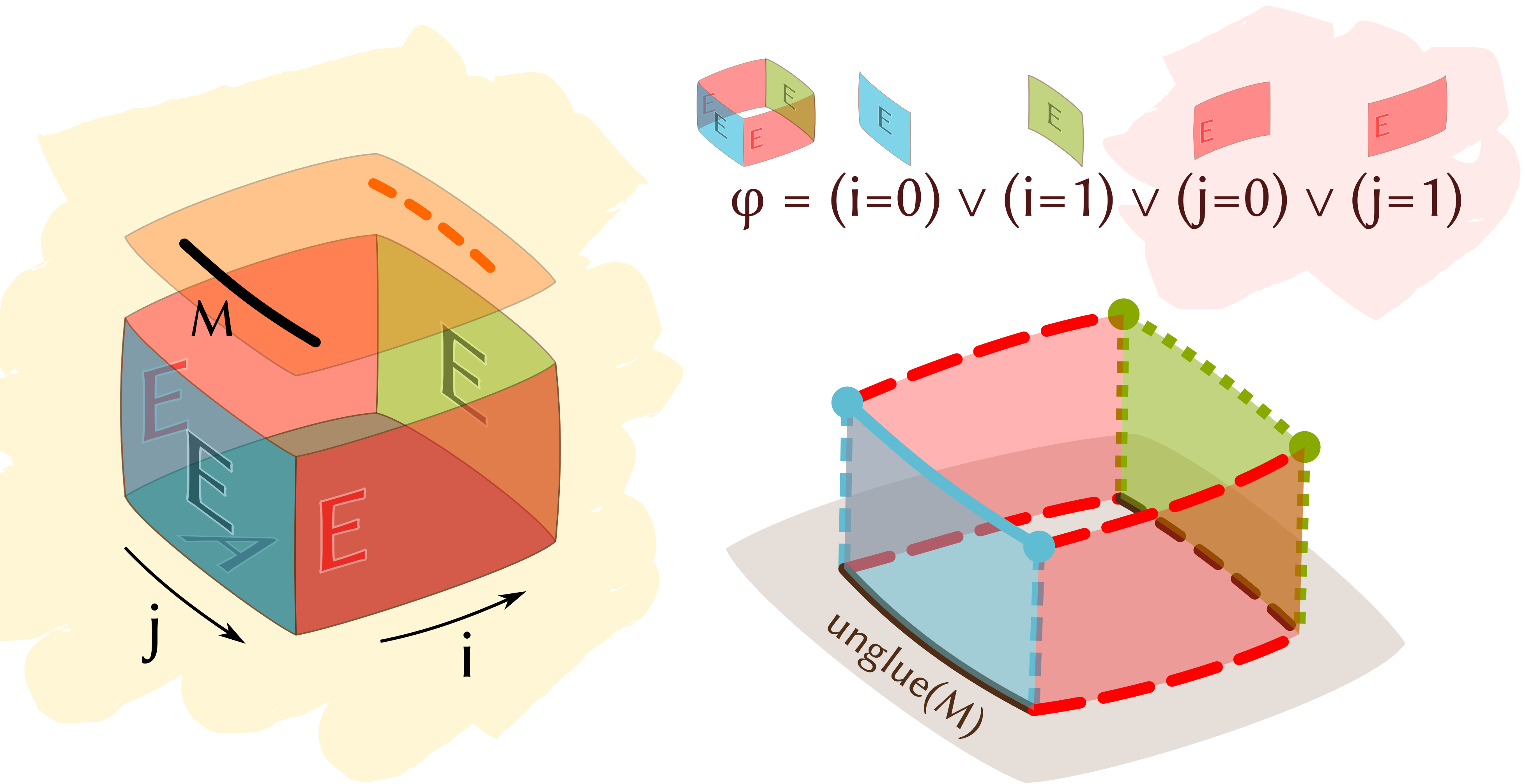
## Univalence

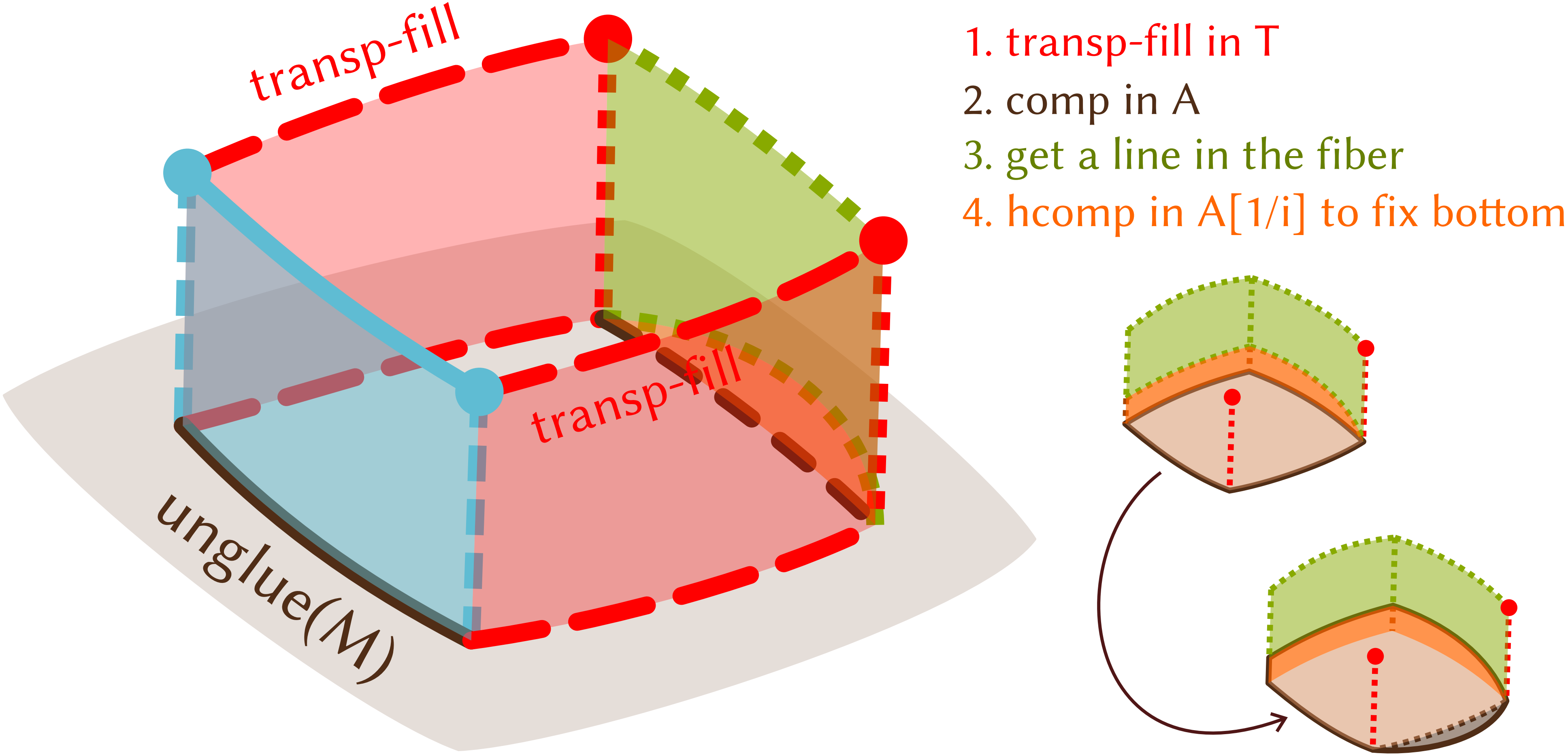


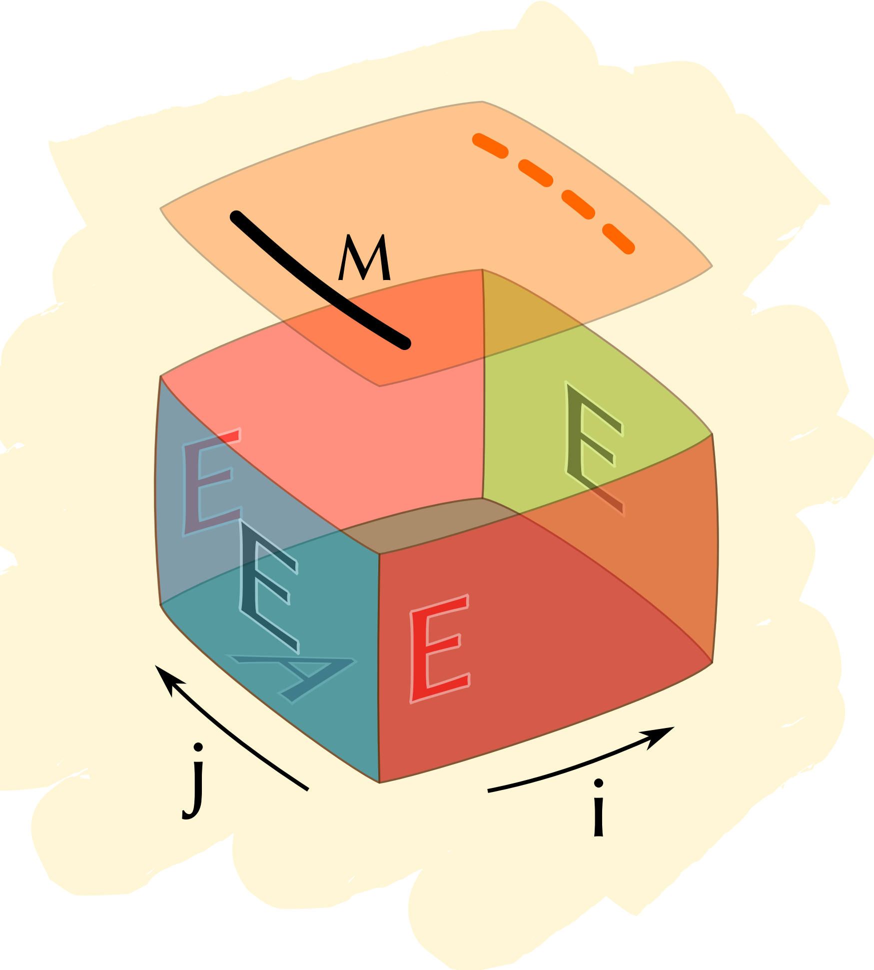


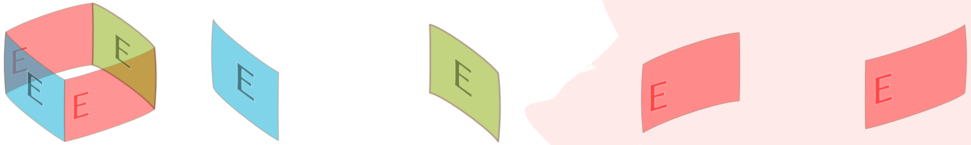

1. Do hfill in T that is consistent with N
2. Calculate unglue(hfill) in A
3. Do the hcomp in A




M
j
i
φ = (i=0) ∨ (i=1) ∨ (j=0) ∨ (j=1)
unglue(M)


transp-fill
transp-fill
unglue(M)
1. transp-fill in T
2. comp in A
3. get a line in the fiber
4. hcomp in A[1/i] to fix bottom




$$\varphi = (i=0) \lor (i=1) \lor (j=0) \lor (j=1)$$

$$\forall i.\varphi$$

deleting any equation with i

$$\forall i.\varphi = \text{false} \lor \text{false} \lor (j=0) \lor (j=1)$$

Further optimizations to reduce hcomp

Implemented in cubical Agda

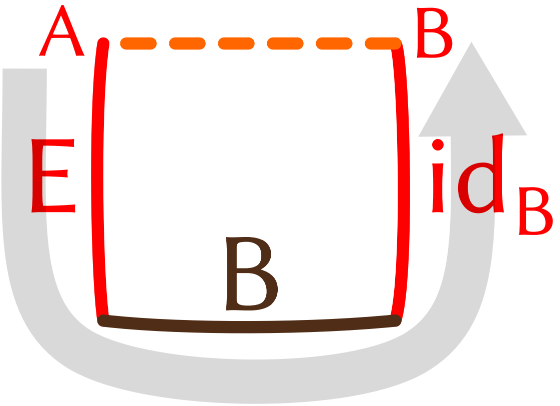
How about $\text{transp}^i$ (Glue ...) $\psi$ a?

Add constraints to all steps

(not 0)

☛ check the references on the course website

# Univalence (optimized)




$$\mathrm{transp}^i\ (\mathrm{ua}(E)@i)\ 0\ a$$
$$\equiv \mathrm{id}\ (\mathrm{transp}^i\ B\ 0\ (\mathrm{fst}\ E\ a))$$
$$\not\equiv \mathrm{fst}\ E\ a$$

coercion along constant types is not identity
and it seems difficult to add this equation

*(called "regularity")*

PS: coercion in certain types are identity

# Congratulations!

We have a cubical & univalent dependent type theory!

*NEXT: meta-theory*