Correction:

We will redo the empty type

*the rules were wrong; the video was re-uploaded*
\( \text{hcomp}^i (\prod_{x:A} B) [\varphi \mapsto N] M \)

\( \lambda x. \text{hcomp}^i B [\varphi \mapsto N(x)] M(x) \)

*fill the square with the same \( x \) along \( i \)*
transp^i (\Pi_{x:A} B) \varphi M

\downarrow

\lambda x.\text{transp}^i B[^{\text{filler}^i(x)/x}] \varphi M(\text{filler}^0(x))

\text{filler}^i(x) \equiv \text{transp-fill}^\sim^i A \varphi x

back and forth
hcomp^i (Σ_{x:A} B) [φ ↦ N] M

\[ \langle \text{hcomp}^i A [φ ↦ \text{fst}(N)] \text{fst}(M), \text{compi} B[\text{filleri}/x] [φ ↦ \text{snd}(N)] \text{snd}(M) \rangle \]

filleri : \equiv \text{hfill}^i A [φ ↦ \text{fst}(N)] \text{fst}(M)

**needs compi^i because**

\( B(\text{filler}/x) \) **depends on** \( i \)
\[ \text{transp}^i (\Sigma_{x:A} B) \varphi M \]
\[
\langle \text{transp}^i A \varphi \text{fst}(M), \text{transp}^i B[\text{filler}^i/x] \varphi \text{snd}(M) \rangle \\
\text{filler}^i \equiv \text{transp-fill}^i A \varphi \text{fst}(M) \]
DERIVED OPERATORS

\[ \text{transp-fill}^i A \varphi M \]

\[ \text{hfill}^i A [\varphi \mapsto N] M \]

\[ \text{comp}^i A [\varphi \mapsto N] M \]
the unit
functions
pairs
paths
natural numbers
disjoint sums
the empty type
the circle
universes
\[ hcomp^i (\text{Path}_j.A(M; N)) [\varphi \mapsto Q] P \]
\[ \equiv \lambda j. hcomp^i A [\varphi \mapsto Q@j, j=0 \mapsto M, j=1 \mapsto N] P@j \]
\[ \text{transp}^i (\text{Path}_{j,A}(M; N)) \varphi P \]
\[ \equiv \lambda j. \text{comp}^i A [\varphi \mapsto P@j, j=0 \mapsto M, j=1 \mapsto N] \ P@j \]
- the unit
- functions
- pairs
- paths
- negative types
- natural numbers
- disjoint sums
- the empty type
- the circle
- universes
- positive types
$M$ always works
Freely generated inductive types now have irreducible hcomps

If $\varphi = \text{true}$, this should reduce to $N[1/i]$

Otherwise, what should we do?
elim(hcomp) = comp(elim)

\[ E(h\text{compi}_N[\varphi \mapsto O]P) \equiv \text{compi}_C[\text{filleri}^i/x][\varphi \mapsto E(O)]E(P) \]

\[ E(O) \equiv \text{elim}_N[x.C](M; x.y.N; O) \]

\[ \text{filleri}^i \equiv h\text{filli}_N[\varphi \mapsto O]P \]
1. Inductive types have formal hcomps
2. Elim commutes with formal hcomps

e.g., ⊥, ℕ, 2, A+B, the circle, etc.

transp can always be reduced*
*except for indexed inductive families
\[
\text{transp} \ (A+B) \ \phi \ \text{inl}(M) \equiv \ \text{inl}(\text{transp} \ A \ \phi \ M) \\
\text{transp} \ (A+B) \ \phi \ \text{inr}(M) \equiv \ \text{inr}(\text{transp} \ B \ \phi \ M)
\]
Optional*  

**hcomp can commute with constructors**

\[
\begin{align*}
N && \text{suc}(\text{hcomp}^i N [\varphi \mapsto N] M) &\equiv \text{hcomp}^i N [\varphi \mapsto \text{suc}(N)] \text{suc}(M) \\
A+B && \text{inl}(\text{hcomp}^i A [\varphi \mapsto N] M) &\equiv \text{hcomp}^i (A+B) [\varphi \mapsto \text{inl}(N)] \text{inl}(M) \\
&& \text{inr}(\text{hcomp}^i B [\varphi \mapsto N] M) &\equiv \text{hcomp}^i (A+B) [\varphi \mapsto \text{inr}(N)] \text{inr}(M)
\end{align*}
\]

*Cubical Agda has these rules*