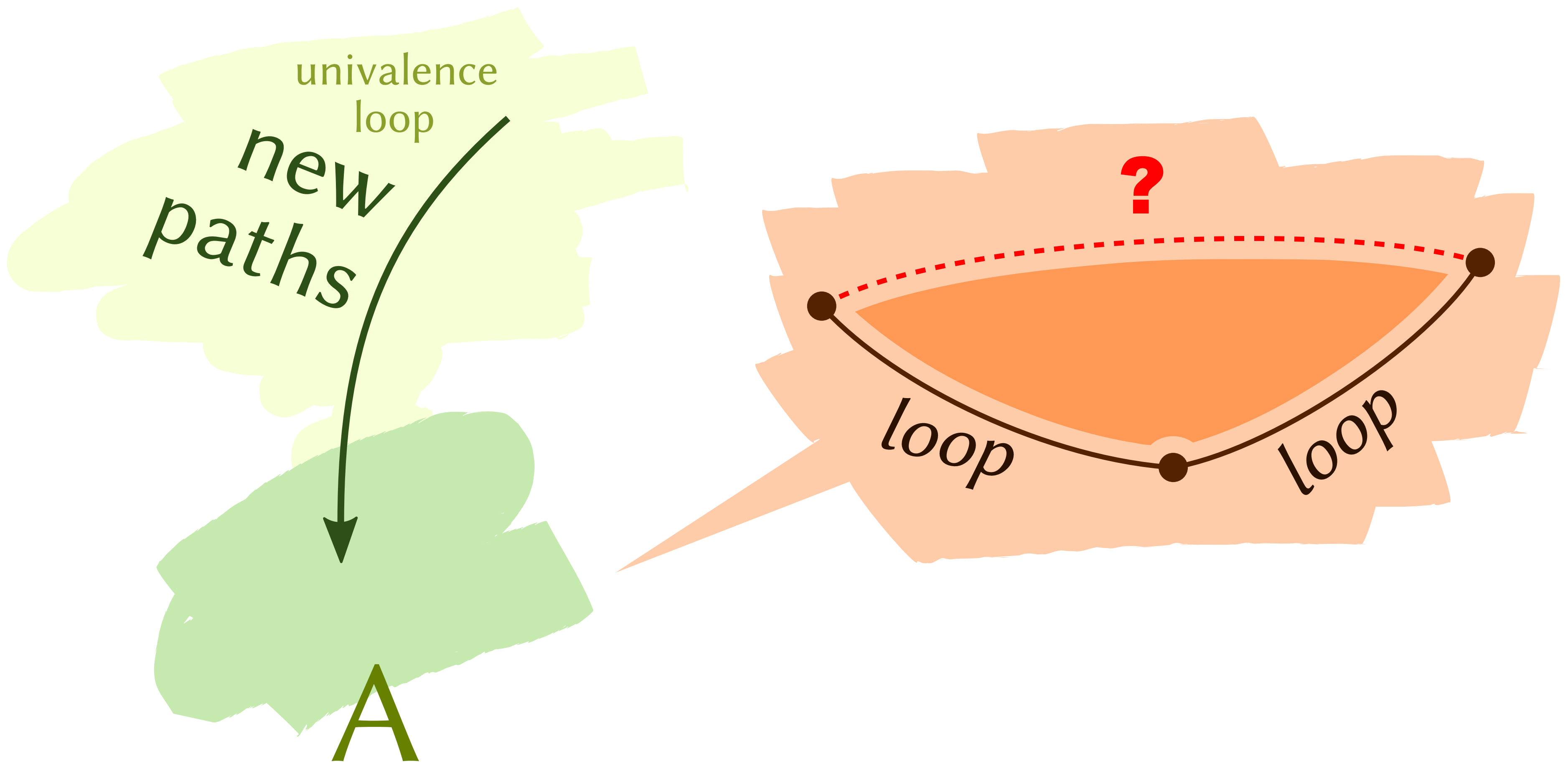
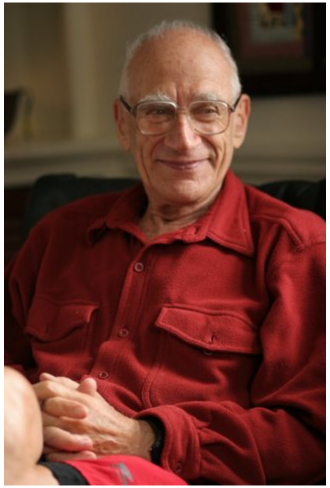
BAYE
CUCLTES

PART II


univalence
loop
new
paths
A
?
loop
loop



Daniel Marinus Kan

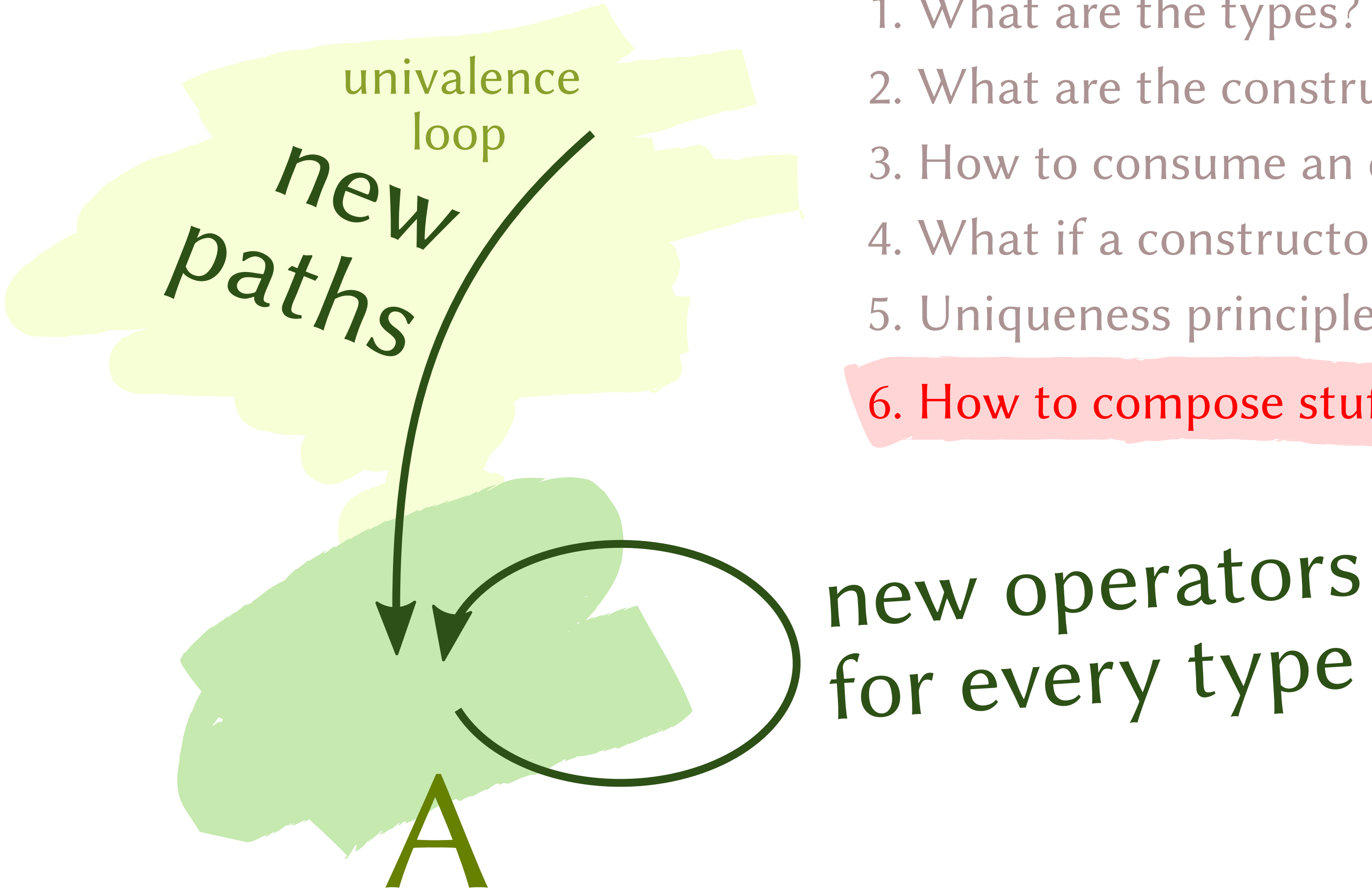
I know how to guarantee a combinatorial structure has enough paths

My group knows how to bring that into the design of type theory

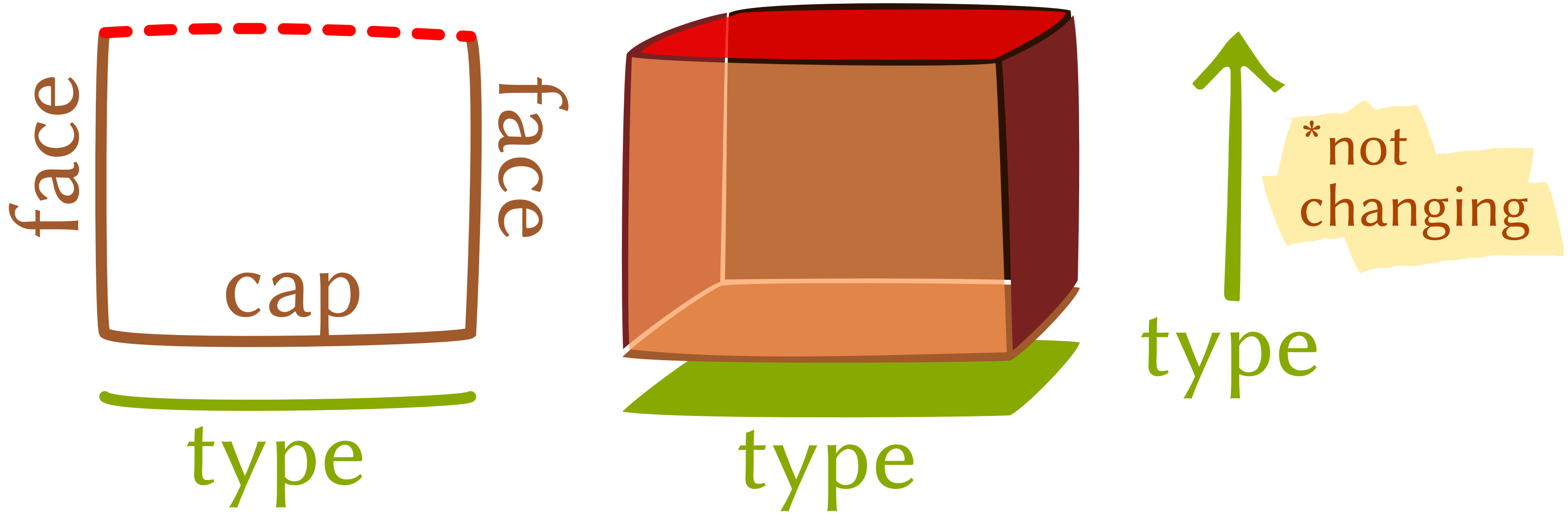
*Guillaume Brunerie and Daniel R. Licata are also pioneers

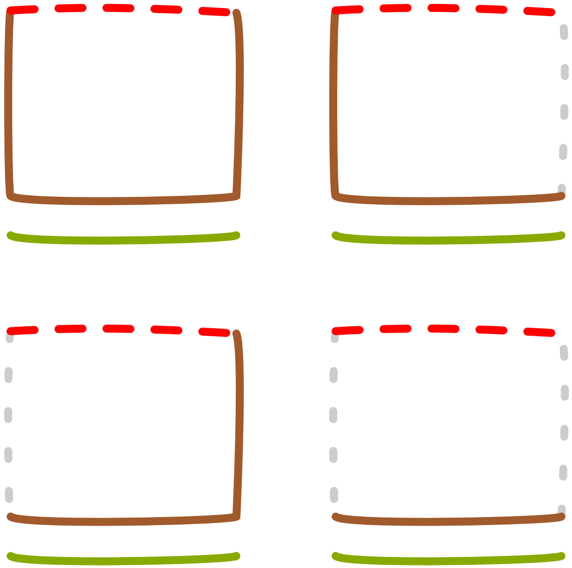
Thierry Coquand

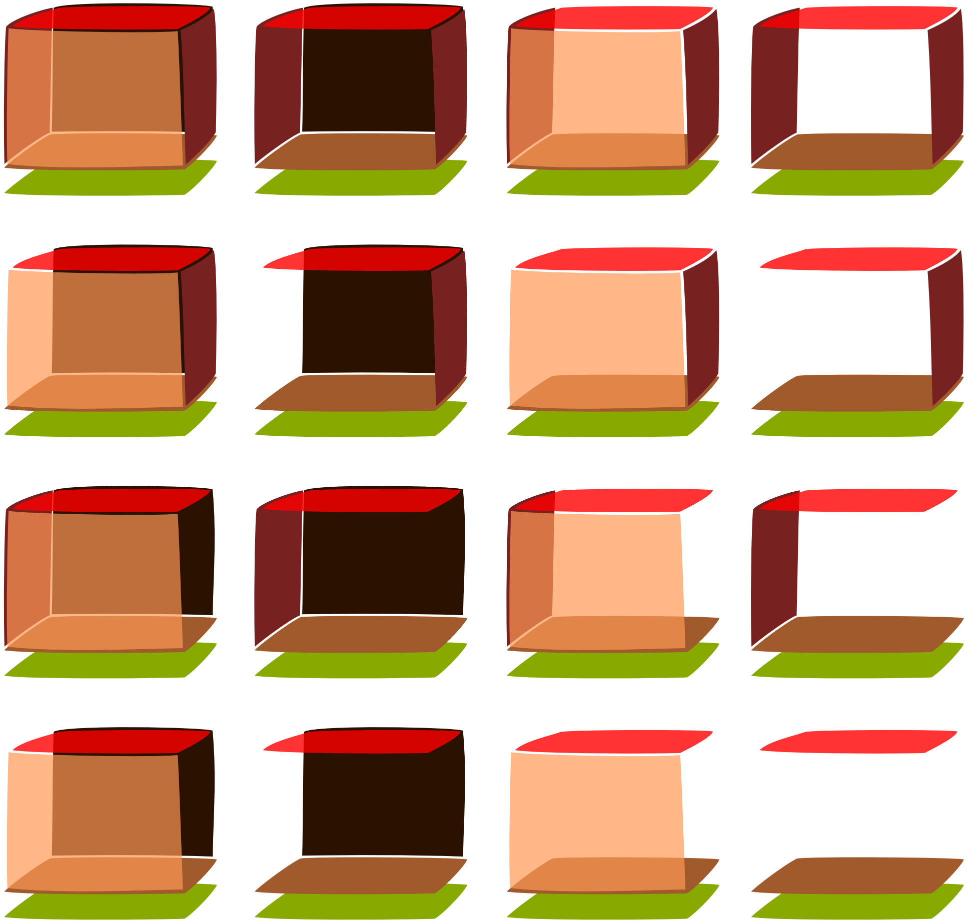
1. What are the types? (form)
2. What are the constructors? (intro)
3. How to consume an element? (elim)
4. What if a constructor is consumed? (β)
5. Uniqueness principle? (η)
6. How to compose stuff? (Kan operators)



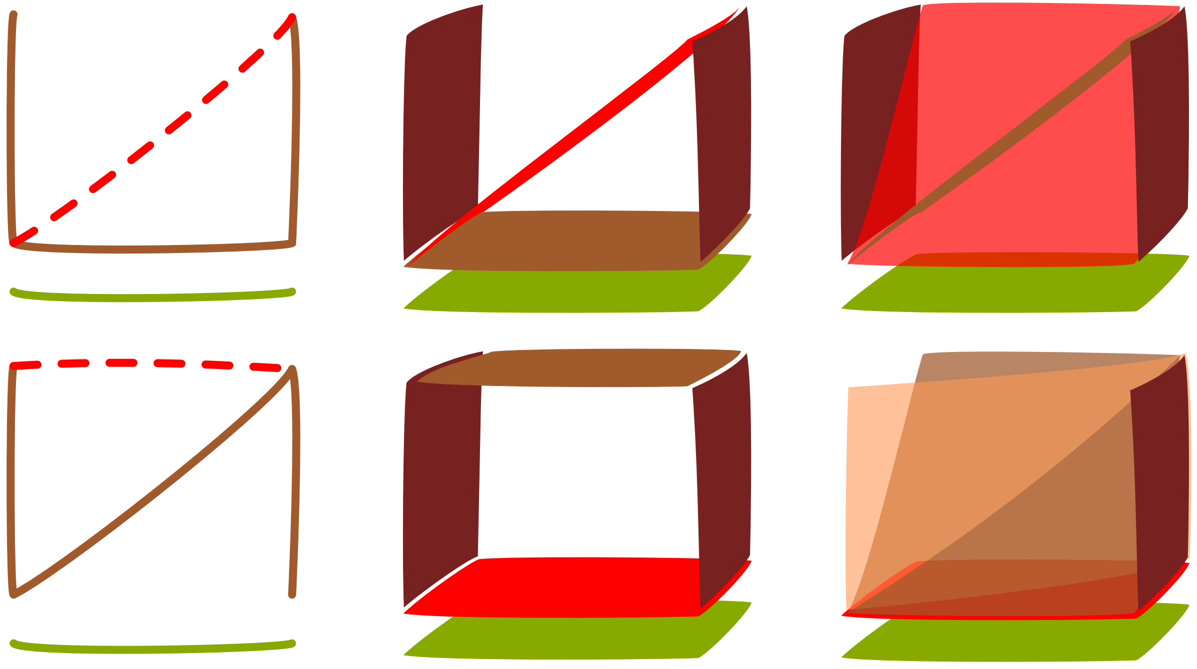
*Homogeneous Compositions





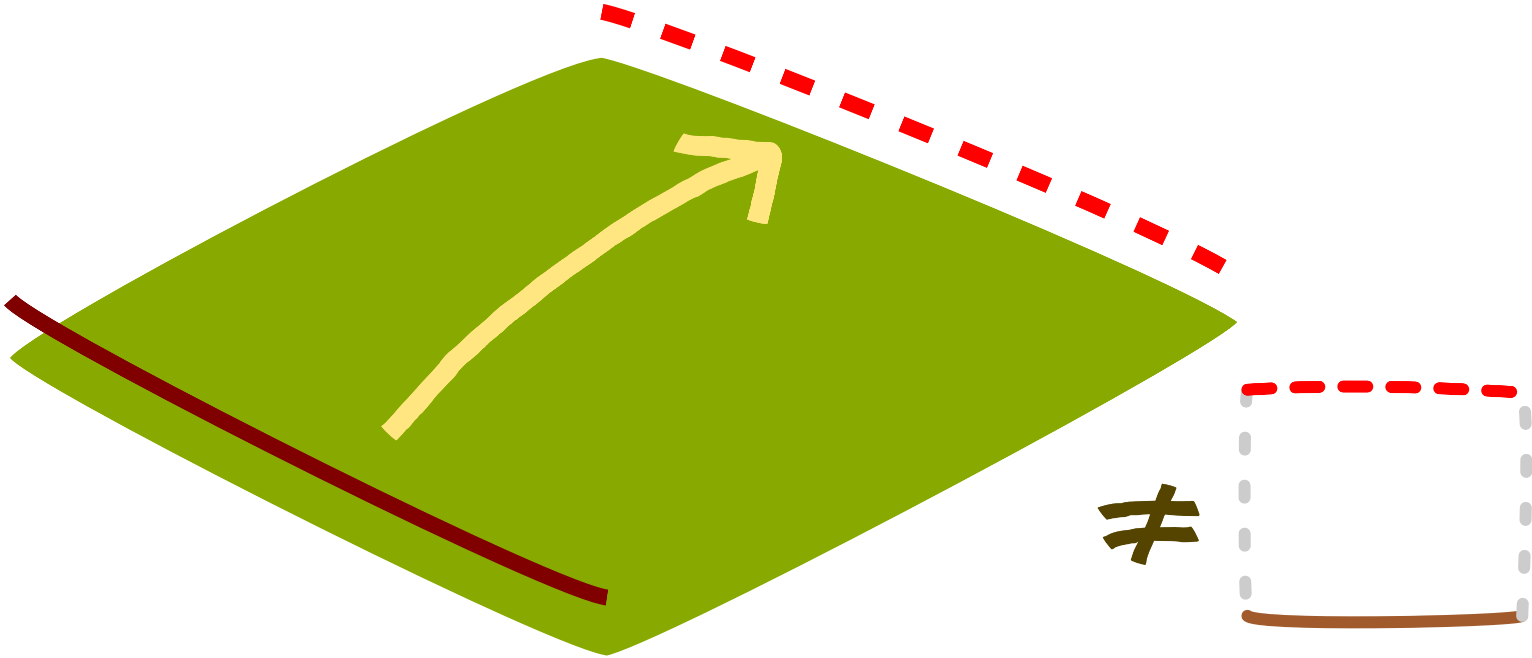


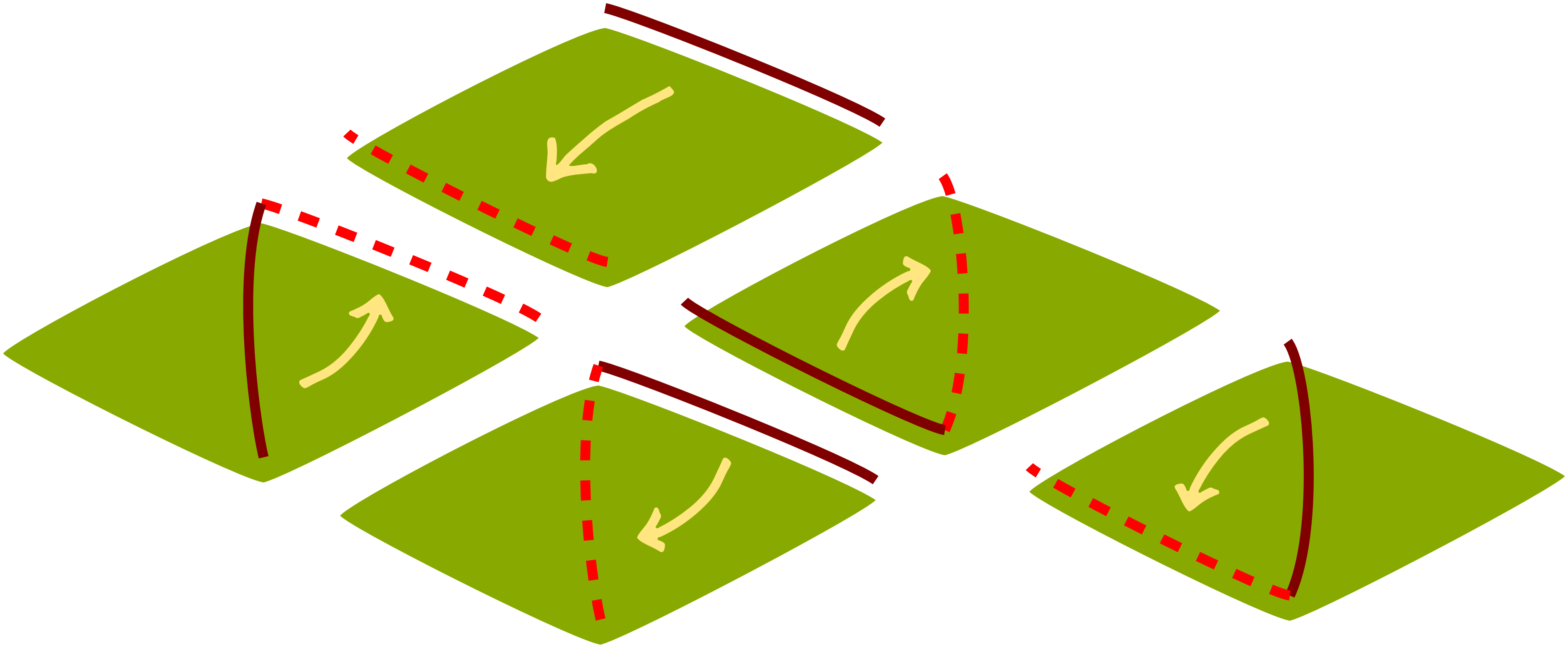
should work with substitution [BCH]



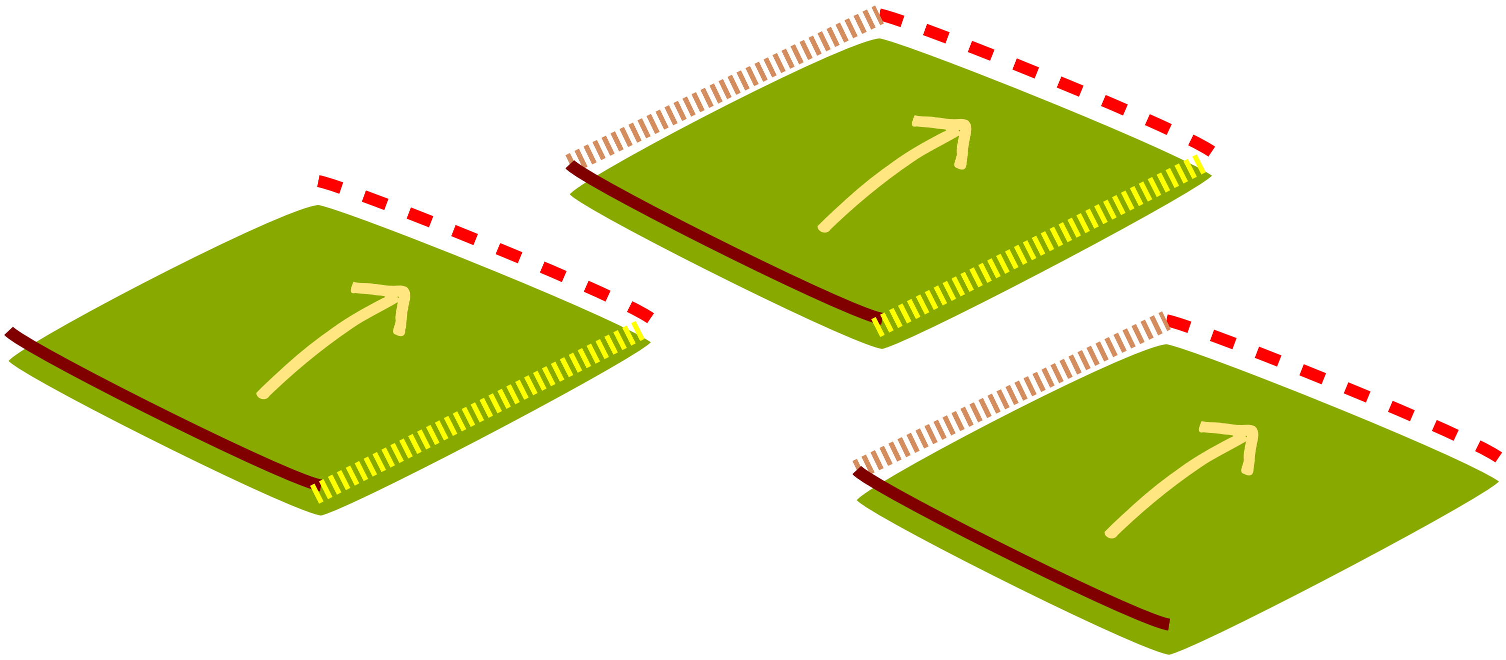
variant: diagonal faces and alternative filling directions

Coercion



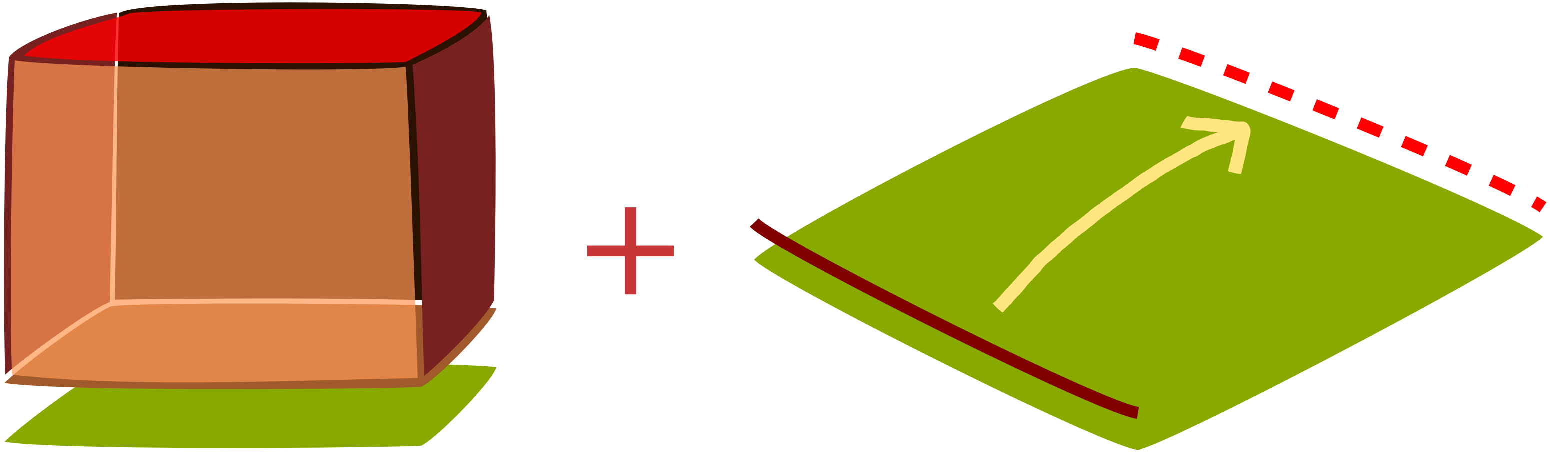


variant 1: alternative coercion directions

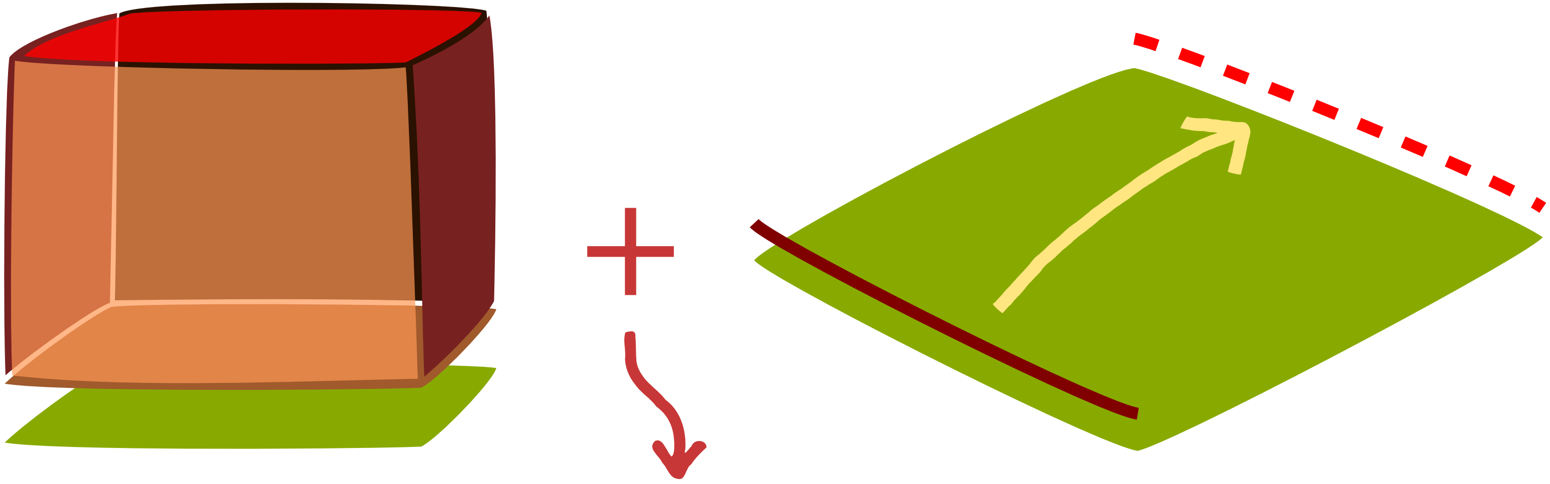


variant 2: freezing parts of the input

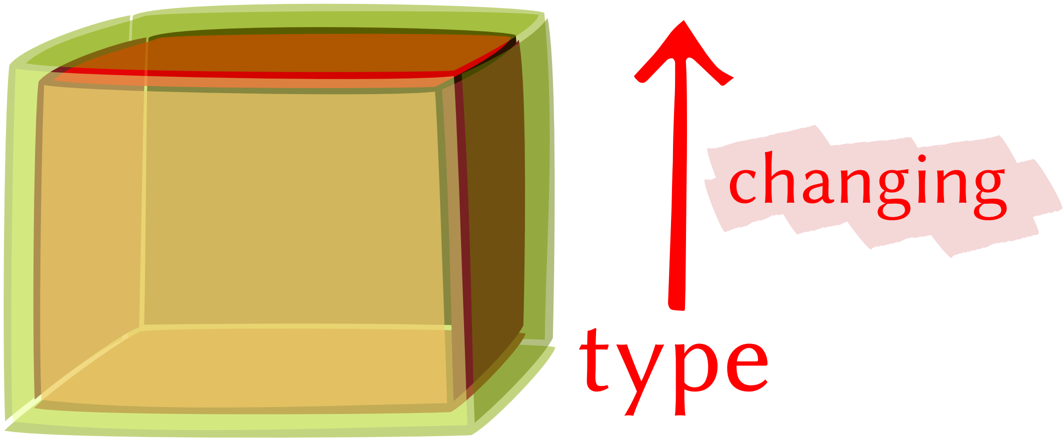
(used in cubical Agda)



With these two operators
every type has enough paths

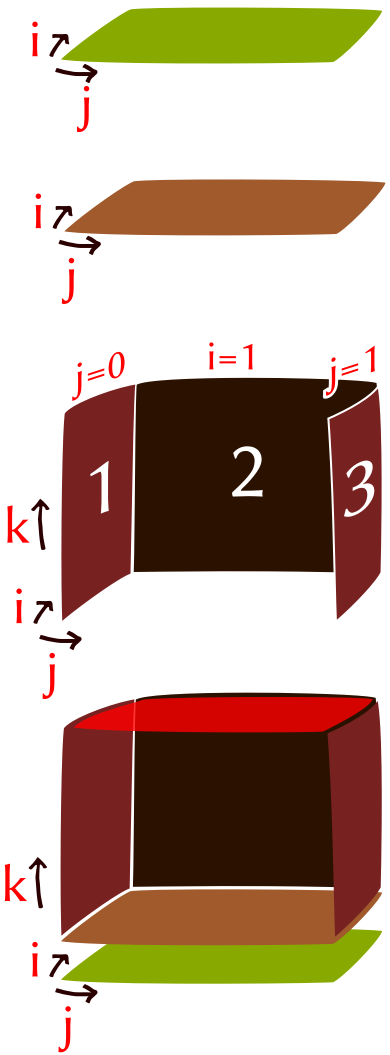


They also give heterogeneous compositions



Major Variants

	[CCHM+CHM]	[AFH+ABCFHL+CH]
algebra on $\mathbb{I}$	$0, 1, \wedge, \vee, \sim$ De Morgan	0, 1
homogeneous composition	standard	variant
coercion	variant 2	variant 1
ready-to-use proof assistants	cubical Agda	redtt



$A : U$

$M : A$

$j=0,\ k:\mathbb{I} \vdash N_1 : A\ [k=0 \mapsto M]$

$i=1,\ k:\mathbb{I} \vdash N_2 : A\ [k=0 \mapsto M,\ j=0 \mapsto N_1]$

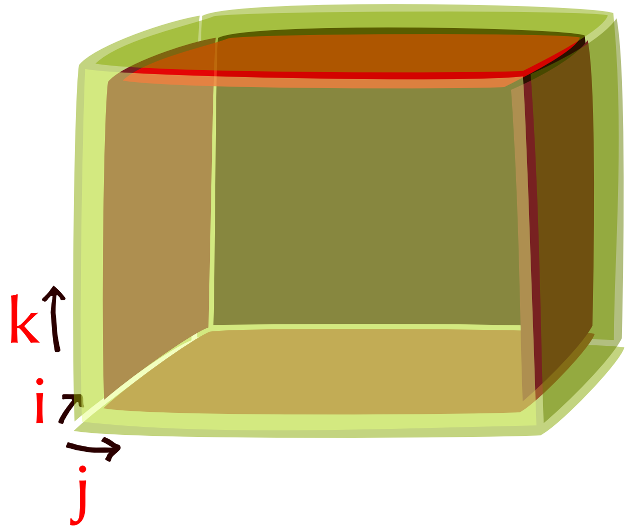
$j=1,\ k:\mathbb{I} \vdash N_3 : A\ [k=0 \mapsto M,\ i=1 \mapsto N_2]$

$[\ j=0 \mapsto N_1,\ i=1 \mapsto N_2,\ j=1 \mapsto N_3\ ]$

(faces are unordered in CCHM+CHM)

$\mathrm{hcomp}^k\ A\ [\ j=0 \mapsto N_1,\ i=1 \mapsto N_2,\ j=1 \mapsto N_3\ ]\ M : A$

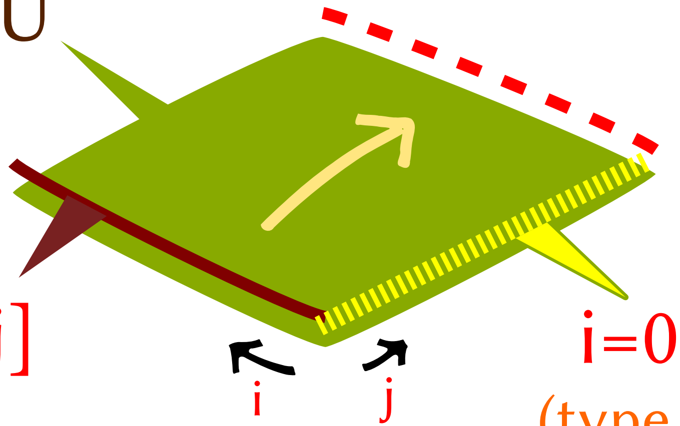
homogeneous



$$k{:}\mathbb{I} \vdash A : U$$

$$M : A[0/k]$$

$$\mathrm{comp}^k\, A\, [\ \ldots\ ]\, M : A[1/k]$$



$$\text{transp}^j\ A\ (\sim i)\ M : A[1/j]$$

this represents (~i=1) = (i=0)

in general, r:𝕀 to represent r=1

Constraints in Contexts

$$\varphi \vdash M : A$$

$$r := 0 \mid 1 \mid i \mid r_1 \wedge r_2 \mid r_1 \vee r_2 \mid \sim r \quad \text{(De Morgan)}$$

$$\varphi := \text{false} \mid \text{true} \mid (r = 0) \mid (r = 1) \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2$$

$r := 0 \mid 1 \mid i \mid r_1 \wedge r_2 \mid r_1 \vee r_2 \mid \sim r$ (De Morgan)

$\varphi := \text{false} \mid \text{true} \mid (r = 0) \mid (r = 1) \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2$

$r \mapsto r{=}1$ preserves $\wedge$, $\vee$, and $\sim$ where $\sim r{=}1$ means $r{=}0$

Any φ is equivalent to $r{=}1$ for some r

e.g., $(i{=}0) \vee (i{=}1) = (\sim i{=}1) \vee (i{=}1) = (\sim i \vee i){=}1$

e.g., $\text{trapns}^j\ A\ r\ M$

Restricted by Partial Elements

$$M : A\,[\varphi \mapsto N]$$

$$M : A \textit{ and } \varphi \vdash M \equiv N : A$$

2. What are the constructors? (intro)

3. How to consume an element? (elim)

4. What if a constructor is consumed? (β)

5. Uniqueness principle? (η)

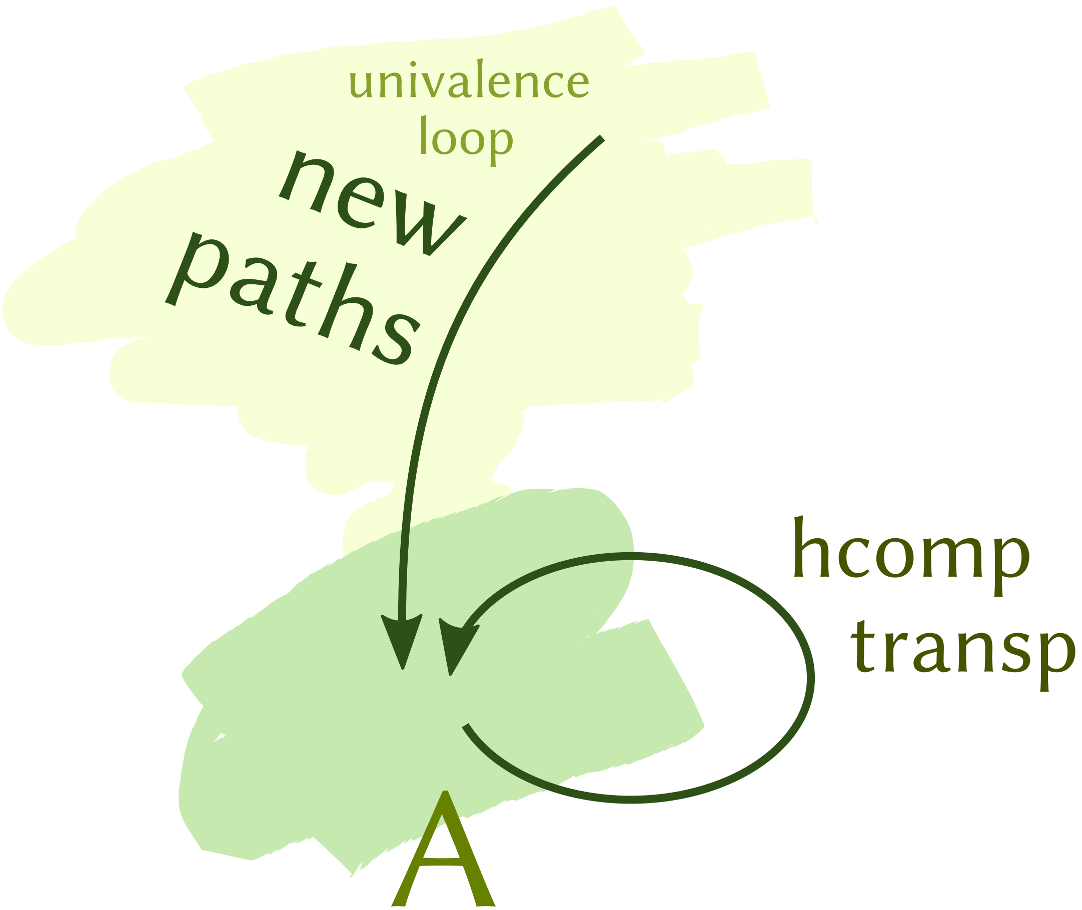
6. hcomp and transp (and thus comp)

✓ Gives us all the paths

✓ Definable for every type

$$\frac{M : T \qquad \varphi, i{:}\mathbb{I} \vdash N : T}{\mathsf{hcomp}^i\ T\ [\varphi \mapsto N]\ M \equiv M : T}$$

$$\frac{M : T \qquad r : \mathbb{I}}{\mathsf{transp}^i\ T\ r\ M \equiv M : T}$$



✓ the unit

the empty type

functions

pairs

paths

the circle

universes

(many others)