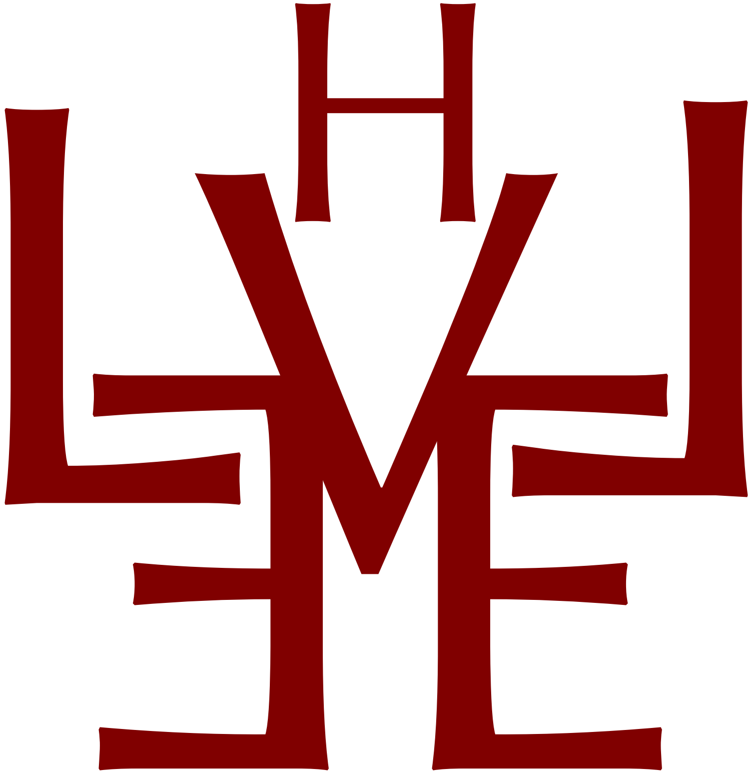
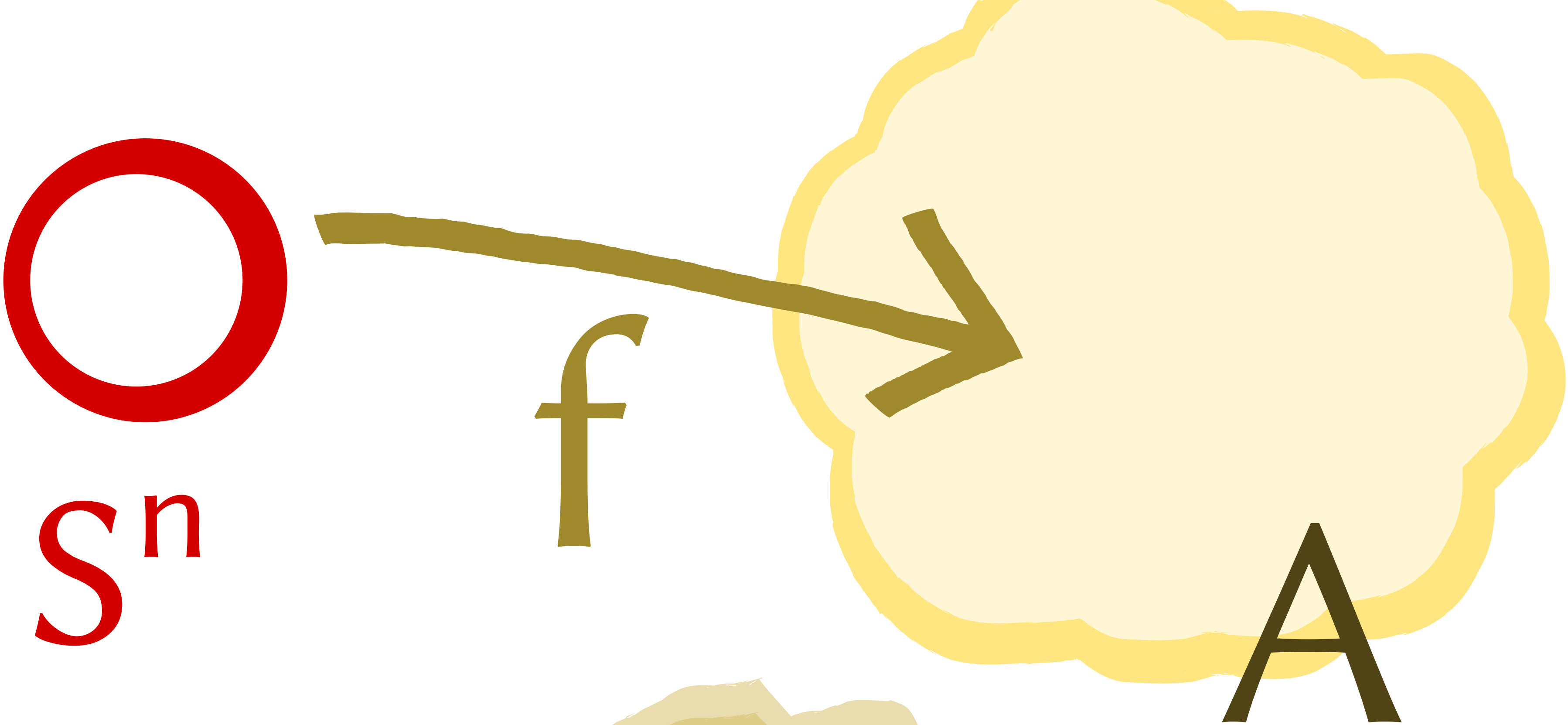
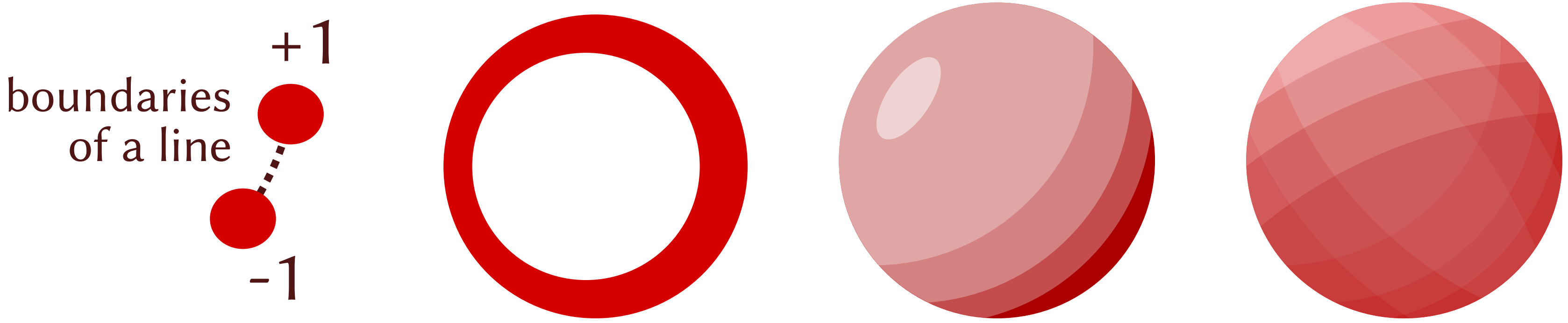




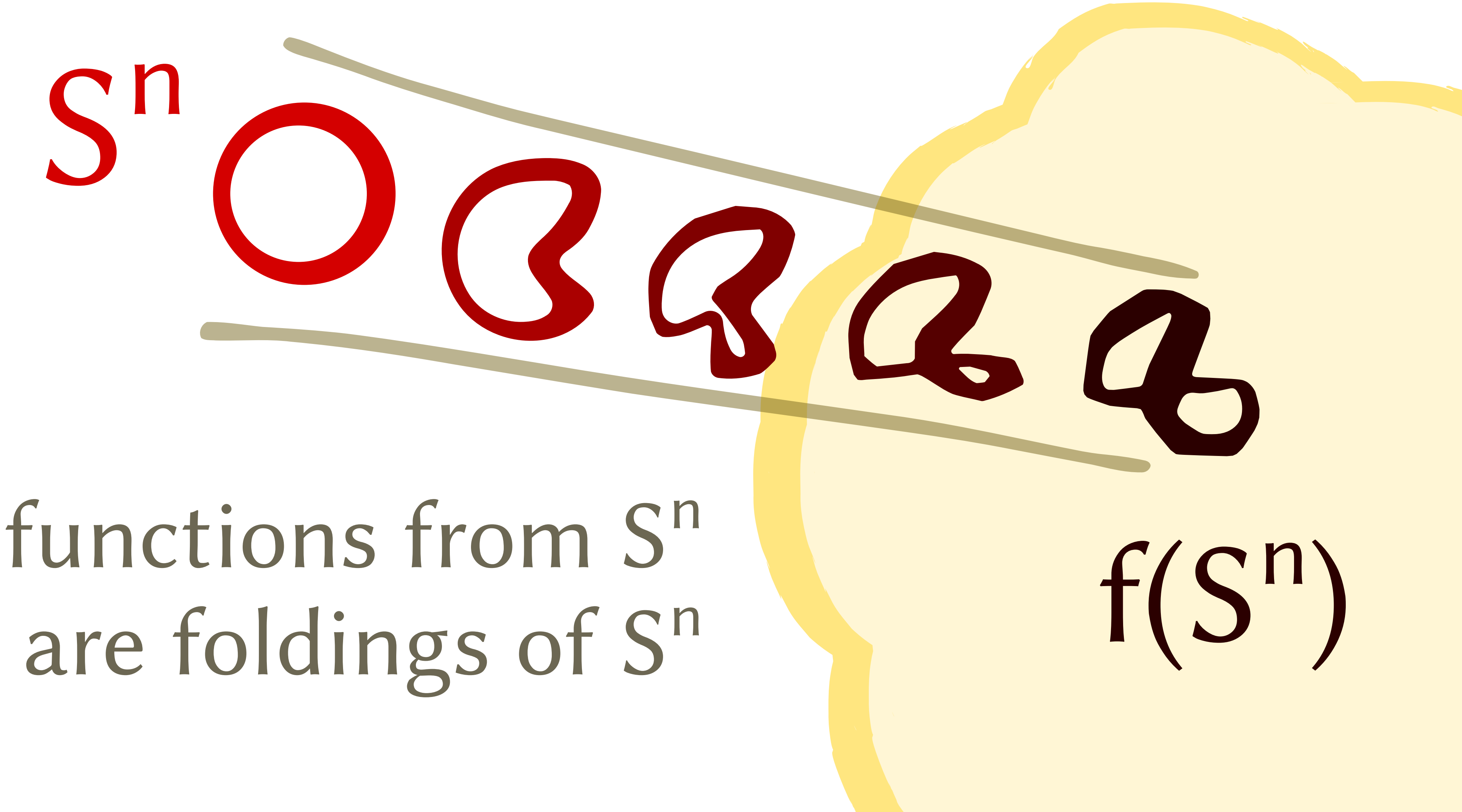
(n-sphere)

functions from S^n to some type



$$S^0 \quad S^1 \quad S^2 \quad S^3$$

points in R^{n+1} of distance 1 from the origin



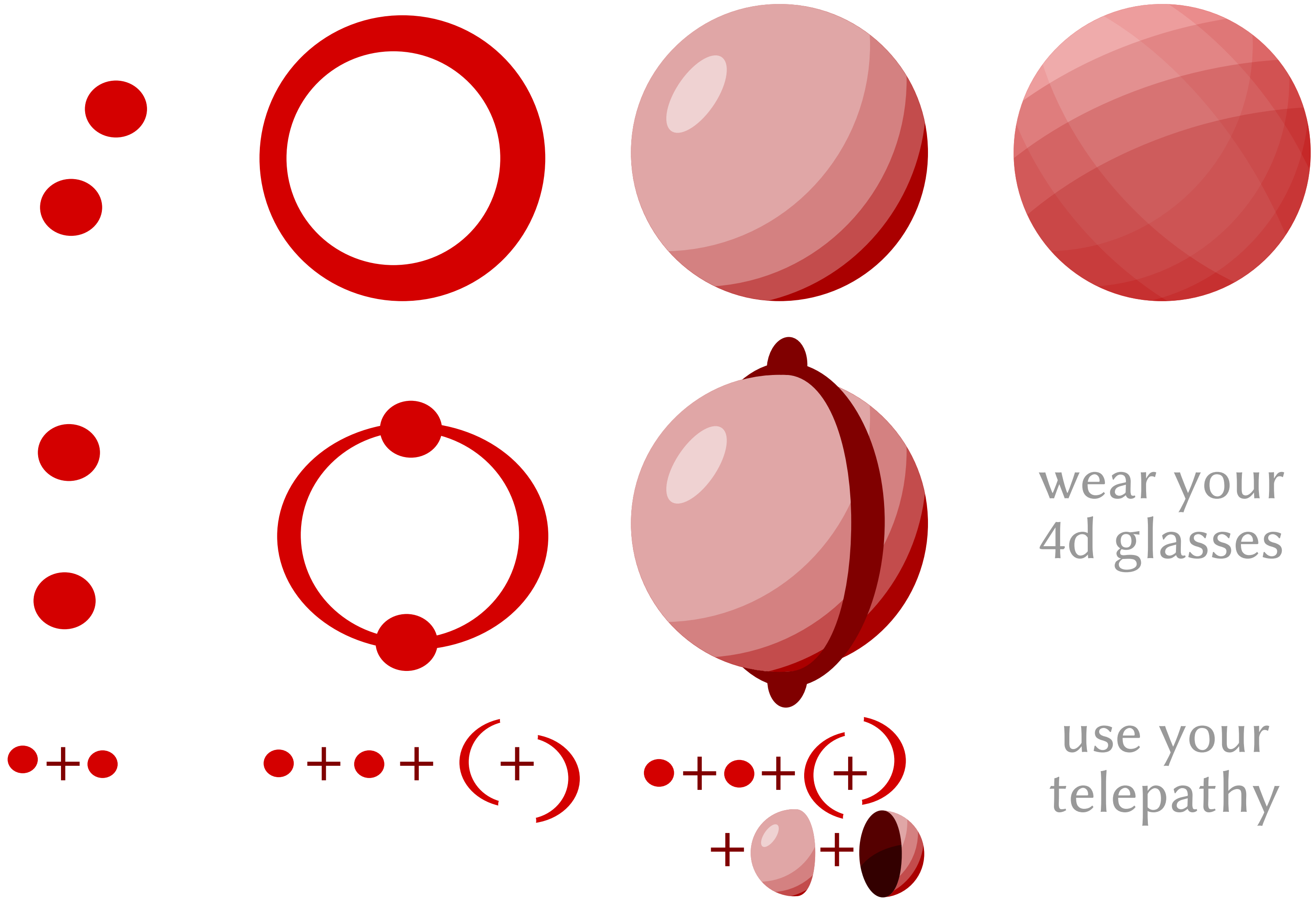
S^n
functions from S^n
are foldings of S^n
$f(S^n)$

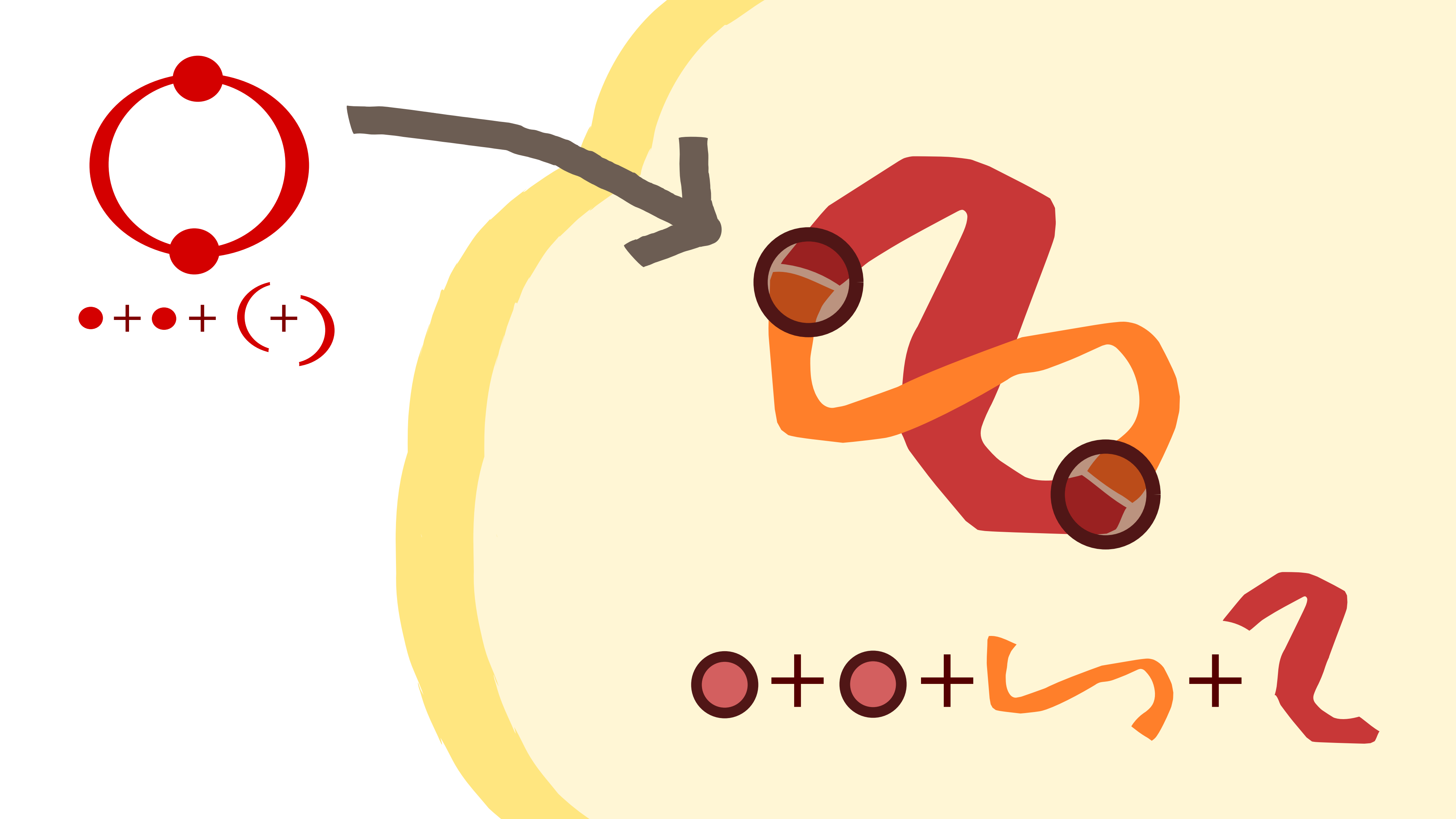
truncation level n

no interesting folding of $S^{m>n}$

no interesting homotopy above dimension n

[Voevodsky]
expressible in type theory!


wear your
4d glasses
use your
telepathy



$$\Pi_{x,y:A}\ \mathrm{Id}(x;y)$$

no interesting foldings of

… or above! (due to continuity)

−1

$$\Pi_{x,y:A}\ \Pi_{p,q:\mathrm{Id}(x;y)}\ \mathrm{Id}(p;q)$$

no interesting foldings of

… or above!

0

$$\Pi_{x,y:A}\ \Pi_{p,q:\mathrm{Id}(x;y)}\ \Pi_{r,s:\mathrm{Id}(p;q)}\ \mathrm{Id}(r;s)$$

no …

of

… or above!

1

$\Pi_{x,y:A}$ Id(x;y) — level -1

$\Pi_{x,y:A}\, \Pi_{p,q:Id(x;y)}$ Id(p;q) — level 0

$\Pi_{x,y:A}\, \Pi_{p,q:Id(x;y)}\, \Pi_{r,s:Id(p;q)}$ Id(r;s) — level 1

$\Pi_{x,y:A}\, \Pi_{p,q:Id(x;y)}\, \Pi_{r,s:Id(p;q)}\, \Pi_{u,v:Id(r;s)}$ Id(u;v) — level 2

$\Pi_{x,y:A}\, \Pi_{p,q:Id(x;y)}\, \Pi_{r,s:Id(p;q)}\, \Pi_{u,v:Id(r;s)}\, \Pi_{a,b:Id(u;v)}$ Id(a;b)

$$\text{has-level}_{n+1}(A) := \Pi_{x,y:A}\text{has-level}_n(\text{Id}_A(x;y))$$

$$\text{has-level}_{-2}(A) := \text{is-contr}(A)^*$$

*is-contr(A) instead of just A to match level -2

$$\text{has-level}_{n+1}(A) := \Pi_{x,y:A}\text{has-level}_n(\text{Id}_A(x;\, y))$$

$$\text{has-level}_{-2}(A) := \text{is-contr}(A)$$

✔ homotopies above n trivial

✓ homotopies in dim 0 trivial

$$\Pi_{x,y:A} \mathrm{Id}(x;y)$$

possibly not continuously

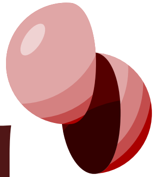
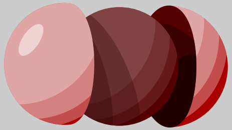
[HoTT 7.3]

✔ homotopies in dim 1 trivial

$$\Pi_{x,y:A}\ \Pi_{p,q:\mathrm{Id}(x;y)}\ \mathrm{Id}(p;q)$$

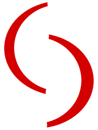
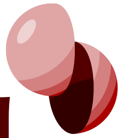
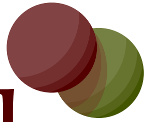
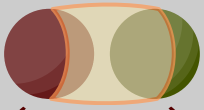
possibly not continuously

✓ homotopies in dim 2 trivial

$\Pi_{x,y:A}\ \Pi_{p,q:\mathrm{Id}(x;y)}\ \Pi_{r,s:\mathrm{Id}(p;q)}$  $\mathrm{Id}(r;s)$ 

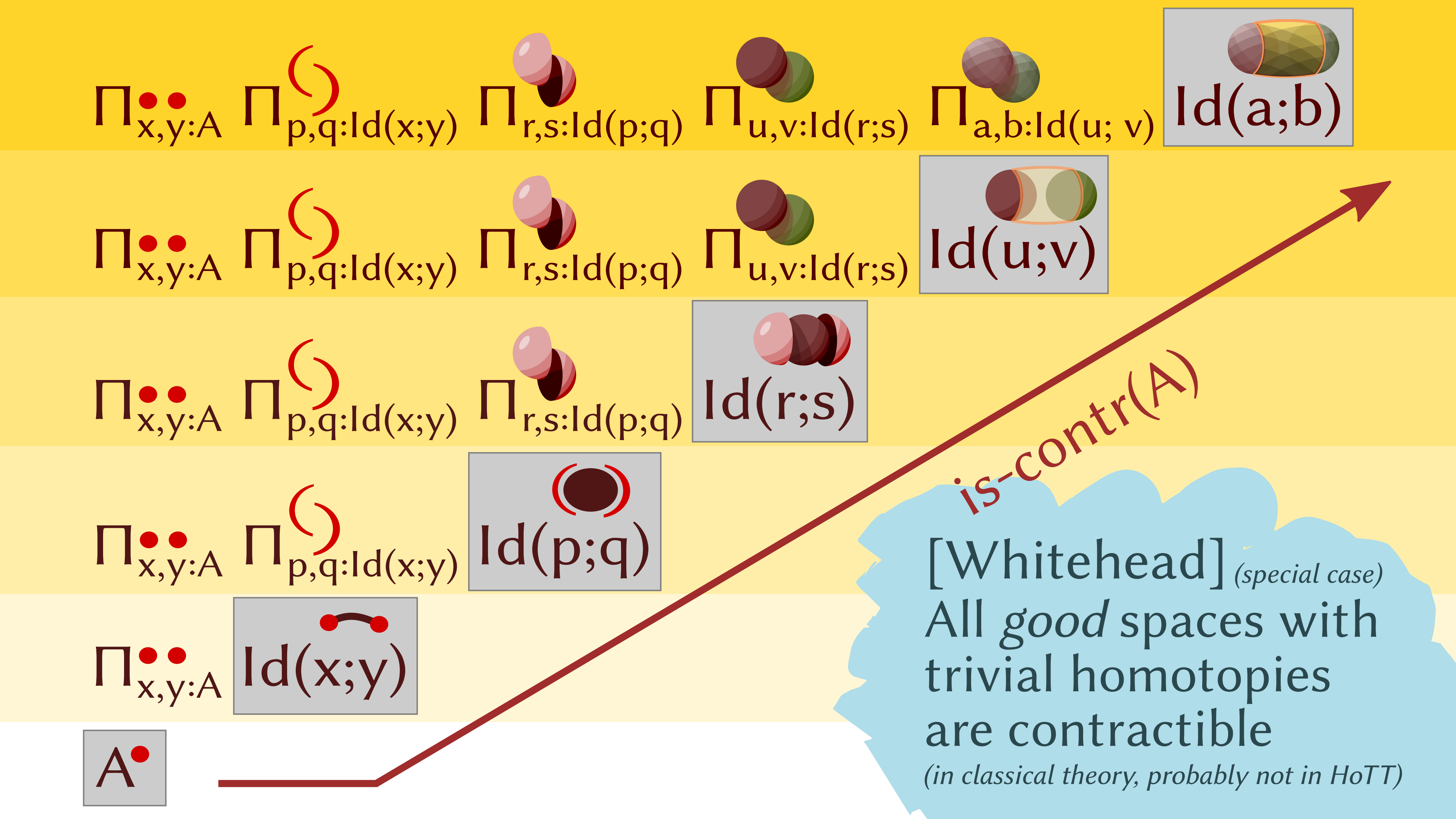
possibly not continuously

✓ homotopies in dim 3 trivial

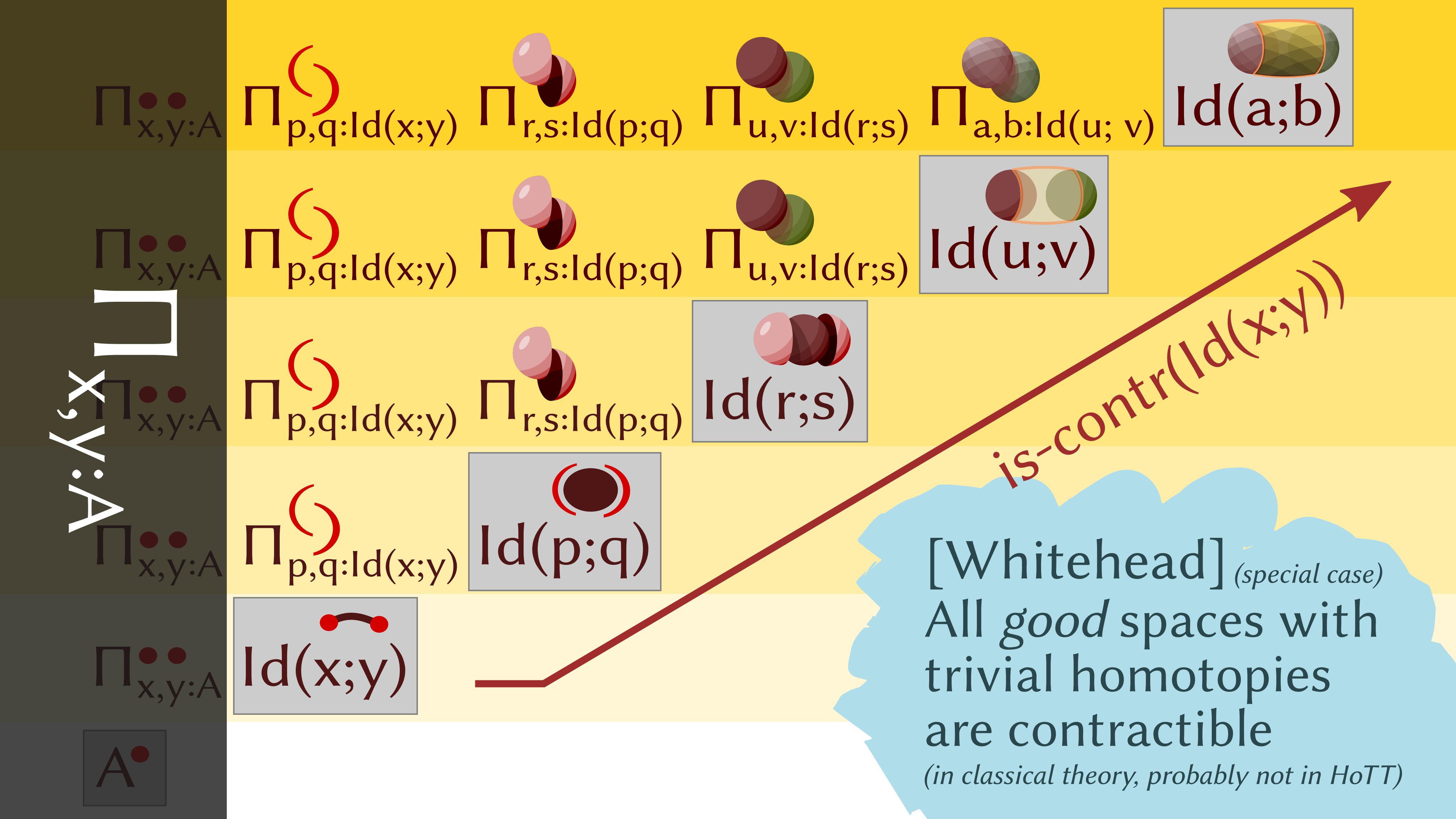
    

$$\Pi_{x,y:A}\ \Pi_{p,q:Id(x;y)}\ \Pi_{r,s:Id(p;q)}\ \Pi_{u,v:Id(r;s)}\ Id(u;v)$$

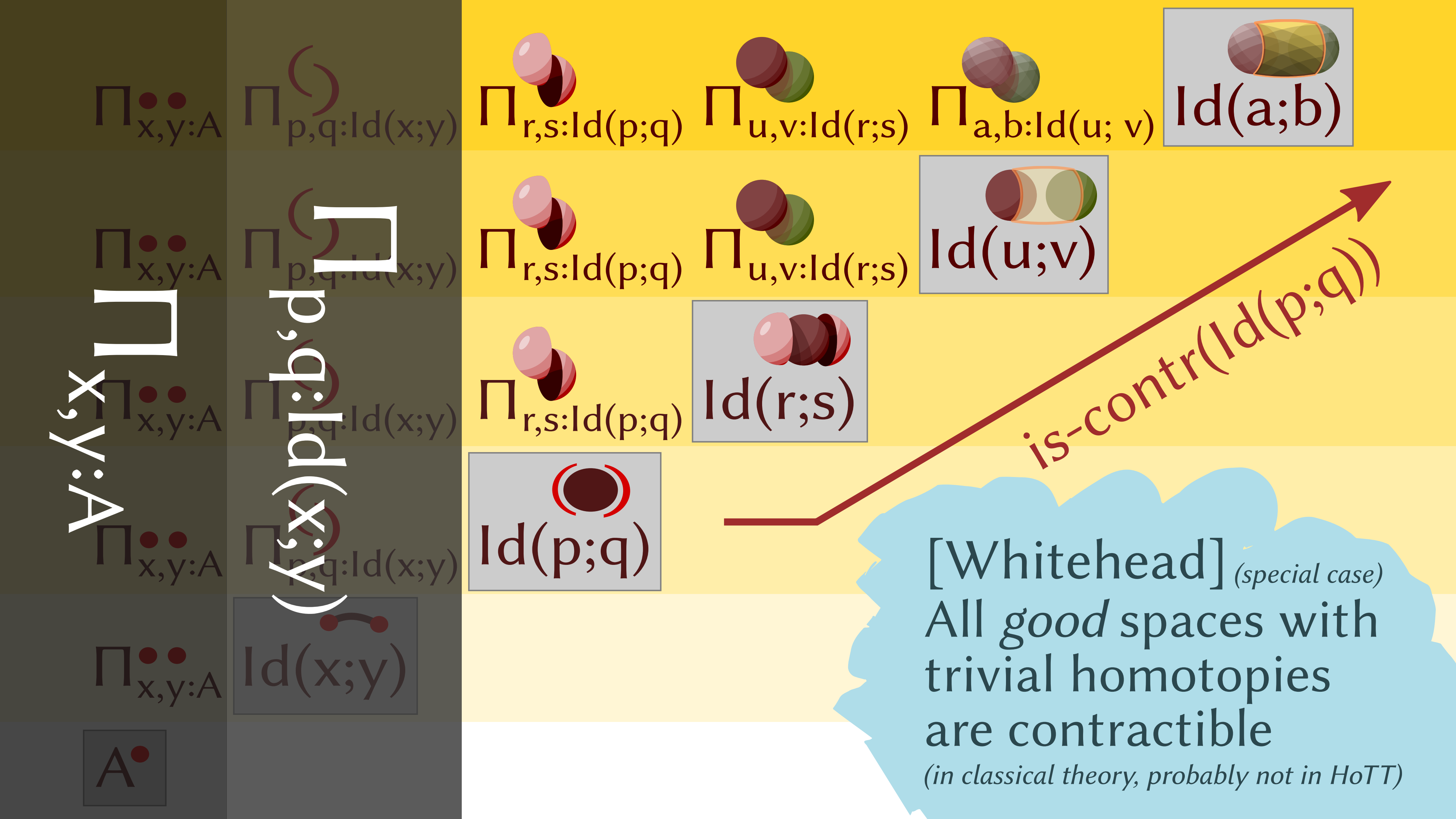
possibly not continuously



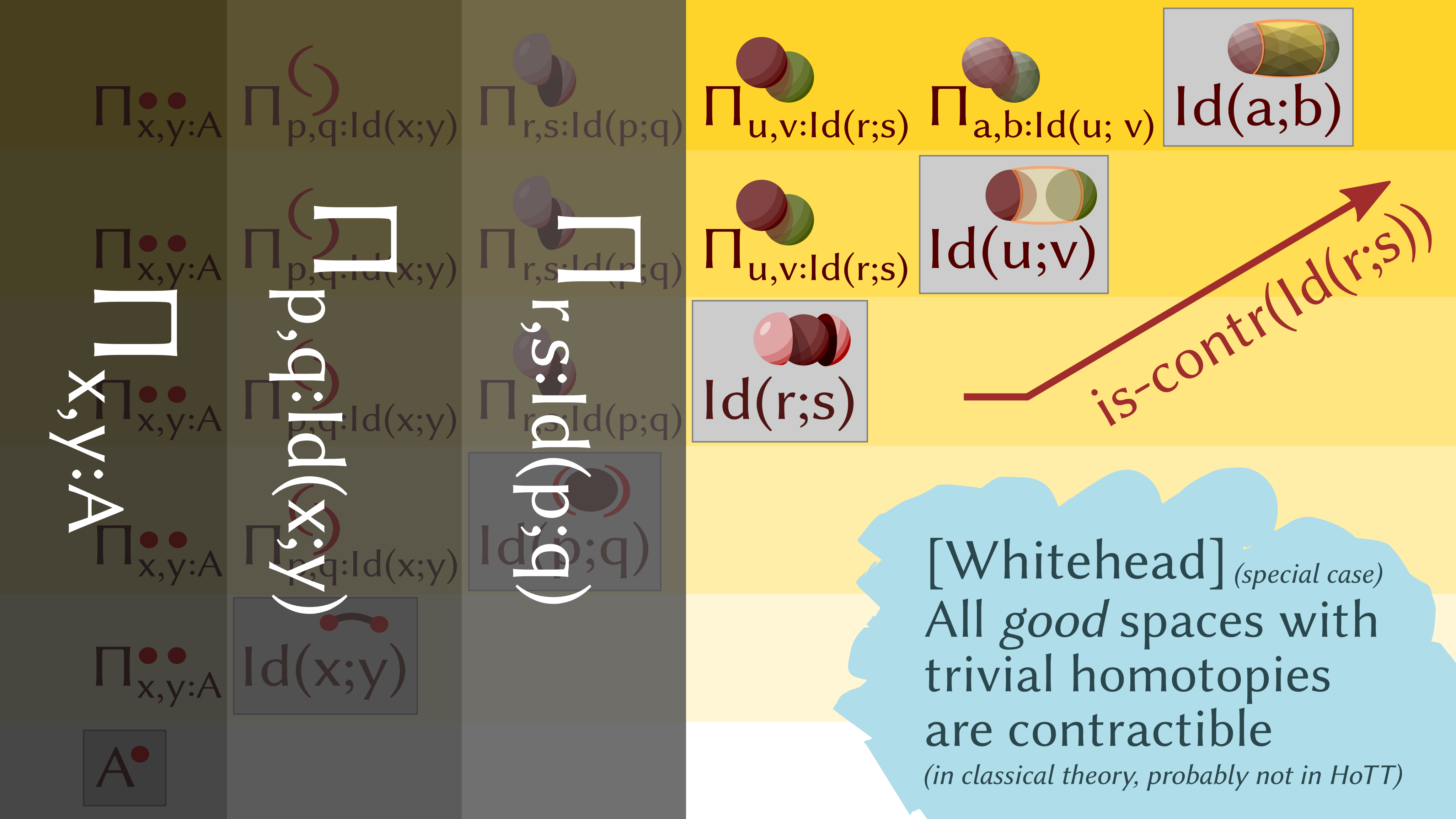
Π x,y:A Π p,q:Id(x;y) Π r,s:Id(p;q) Π u,v:Id(r;s) Π a,b:Id(u; v) Id(a;b)
Π x,y:A Π p,q:Id(x;y) Π r,s:Id(p;q) Π u,v:Id(r;s) Id(u;v)
Π x,y:A Π p,q:Id(x;y) Π r,s:Id(p;q) Id(r;s)
Π x,y:A Π p,q:Id(x;y) Id(p;q)
Π x,y:A Id(x;y)
A
is-contr(A)
[Whitehead] (special case)
All good spaces with trivial homotopies are contractible
(in classical theory, probably not in HoTT)



Π x,y:A
Π x,y:A Π p,q:Id(x;y) Π r,s:Id(p;q) Π u,v:Id(r;s) Π a,b:Id(u; v) Id(a;b)
Π x,y:A Π p,q:Id(x;y) Π r,s:Id(p;q) Π u,v:Id(r;s) Id(u;v)
Π x,y:A Π p,q:Id(x;y) Π r,s:Id(p;q) Id(r;s)
Π x,y:A Π p,q:Id(x;y) Id(p;q)
Π x,y:A Id(x;y)
A
is-contr(Id(x;y))
[Whitehead] (special case)
All good spaces with trivial homotopies are contractible
(in classical theory, probably not in HoTT)



Π x,y:A
Π p,q:Id(x;y)
Π r,s:Id(p;q)
Π u,v:Id(r;s)
Π a,b:Id(u; v)
Id(a;b)
Id(u;v)
Id(r;s)
Id(p;q)
Id(x;y)
A
is-contr(Id(p;q))
[Whitehead] (special case)
All good spaces with
trivial homotopies
are contractible
(in classical theory, probably not in HoTT)



[Whitehead] *(special case)*
All *good* spaces with trivial homotopies are contractible
(in classical theory, probably not in HoTT)

✓ homotopies above -1 trivial

in classical theory

$$\Pi_{x,y:A}\ \text{is-contr}(\mathrm{Id}(x;y))$$

✓ homotopies above 0 trivial

in classical theory

$$\Pi_{x,y:A}\ \Pi_{p,q:\mathrm{Id}(x;y)}\ \text{is-contr}(\mathrm{Id}(p;q))$$

✓ homotopies above 1 trivial

in classical theory

$$\Pi_{x,y:A}\ \Pi_{p,q:\mathrm{Id}(x;y)}\ \Pi_{r,s:\mathrm{Id}(p;q)}\ \text{is-contr}(\mathrm{Id}(r;s))$$

✓ homotopies above 2 trivial

in classical theory

$$\Pi_{x,y:A}\ \Pi_{p,q:\mathrm{Id}(x;y)}\ \Pi_{r,s:\mathrm{Id}(p;q)}\ \Pi_{u,v:\mathrm{Id}(r;s)}\ \text{is-contr}(\mathrm{Id}(u;v))$$

✓ homotopies above n trivial

in classical theory ↓

↑ *in HoTT (and classical theory)*

$$\text{has-level}_{n+1}(A) := \Pi_{x,y:A}\,\text{has-level}_n(\mathrm{Id}_A(x;y))$$

$$\text{has-level}_{-2}(A) := \text{is-contr}(A)$$ ✓