

# Homework 7: Factorization of Weak Constancy

Due 2019/05/12 (Tue) Anywhere on Earth

**Please pay attention to the special deadline 5/12!**

This homework is to finish the exercise we started on 4/20. The theorem is that any weakly constant function from  $A$  to  $B$  factors through the propositional truncation of  $A$  if  $B$  is a set (at truncation level 0). As usual, please email Favonia the completed Agda file.

We say a function  $f$  is weakly constant if it sends any two inputs to the output:

$$\prod_{x_0, x_1 : A} \text{Path}_{\_B}(f\ x_0; f\ x_1)$$

The term “*weak* constancy” was introduced in Kraus et al. [1]. It is in contrast with another notion of “constancy”<sup>1</sup> where you know the value:

$$\sum_{b : B} \prod_{x : A} \text{Path}_{\_B}(f\ a; b)$$

In particular, the identity function of the empty type is weakly constant, but not constant. This may seem odd, but functions involving an empty domain or codomain is always weird.

## Grading

Only one letter grade (without plus or minus) will be assigned to the *entire* homework according to the criterion explained in the syllabus.

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<sup>1</sup>I made a typo in the earlier version.

## References

- [1] Nicolai Kraus et al. *Notions of Anonymous Existence in Martin-Löf Type Theory*. Mar. 2017. doi: 10.23638/LMCS-13(1:15)2017. URL: <https://lmcs.episciences.org/3217>.