redtt

cartesian cubical proof assistant

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oslo, 2018/8/28

joint work with Carlo Angiuli, Evan Cavallo,
Robert Harper, Anders Mörtberg and Jonathan Sterling

type theory

$$\Gamma \vdash A \text{ type}$$

$$\Gamma \vdash A = B \text{ type}$$

$$\Gamma \vdash M : A$$

$$\Gamma \vdash M = N : A$$

cubical type theory

formal intervals $\mathbb{I}$

$$\frac{x:\mathbb{I} \in \Gamma}{\Gamma \vdash x:\mathbb{I}} \qquad \Gamma \vdash 0:\mathbb{I} \qquad \Gamma \vdash 1:\mathbb{I}$$

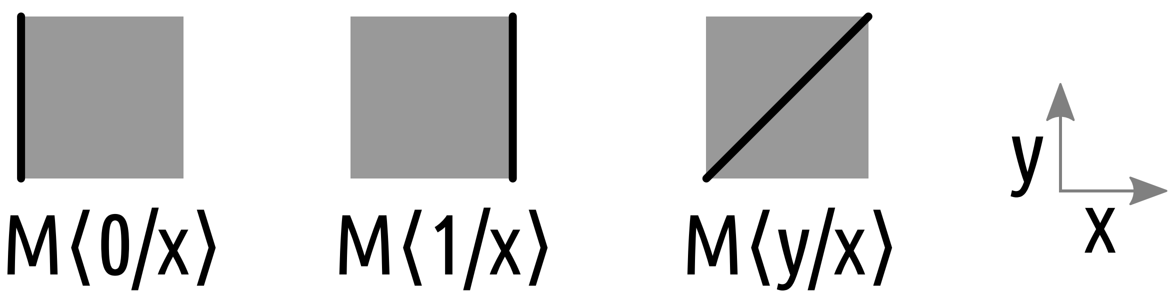
$$\frac{\Gamma \vdash r:\mathbb{I}}{\Gamma \vdash \neg r:\mathbb{I}} \qquad \frac{\Gamma \vdash r:\mathbb{I} \quad \Gamma \vdash s:\mathbb{I}}{\Gamma \vdash r \wedge s:\mathbb{I}} \qquad \frac{\Gamma \vdash r:\mathbb{I} \quad \Gamma \vdash s:\mathbb{I}}{\Gamma \vdash r \vee s:\mathbb{I}}$$

cubical type theory

formal intervals $\mathbb{I}$

$$x_1{:}\mathbb{I},\ x_2{:}\mathbb{I},\ \ldots,\ x_n{:}\mathbb{I} \vdash M : A$$

$$\Leftrightarrow\ M \text{ is an n-cube in } A$$



cubical type theory

formal intervals $\mathbb{I}$

ordinary typing rules hold uniformly

$$\frac{\Gamma, a{:}A \vdash M : B}{\Gamma \vdash \lambda a.M : (a{:}A) \to B}$$

with any number of $\mathbb{I}$ in the Γ

cubical type theory

formal intervals $\mathbb{I}$

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with any number of $\mathbb{I}$ in the Γ

function extensionality due to dimensions commuting with function application

cubical type theory

formal intervals $\mathbb{I}$

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canonicity

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any closed term of $\mathbb{N}$ is equal to some numeral

type-theory tango: internalization of judgmental structure, harmony

cubical type theories

base category

structural rules + operators $\{0, 1, \wedge, \vee, \neg, \ldots\}$

most developed: cartesian, de morgan

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kan structure

cofibrations, fiberwise fibrant replacement

cubical type theories

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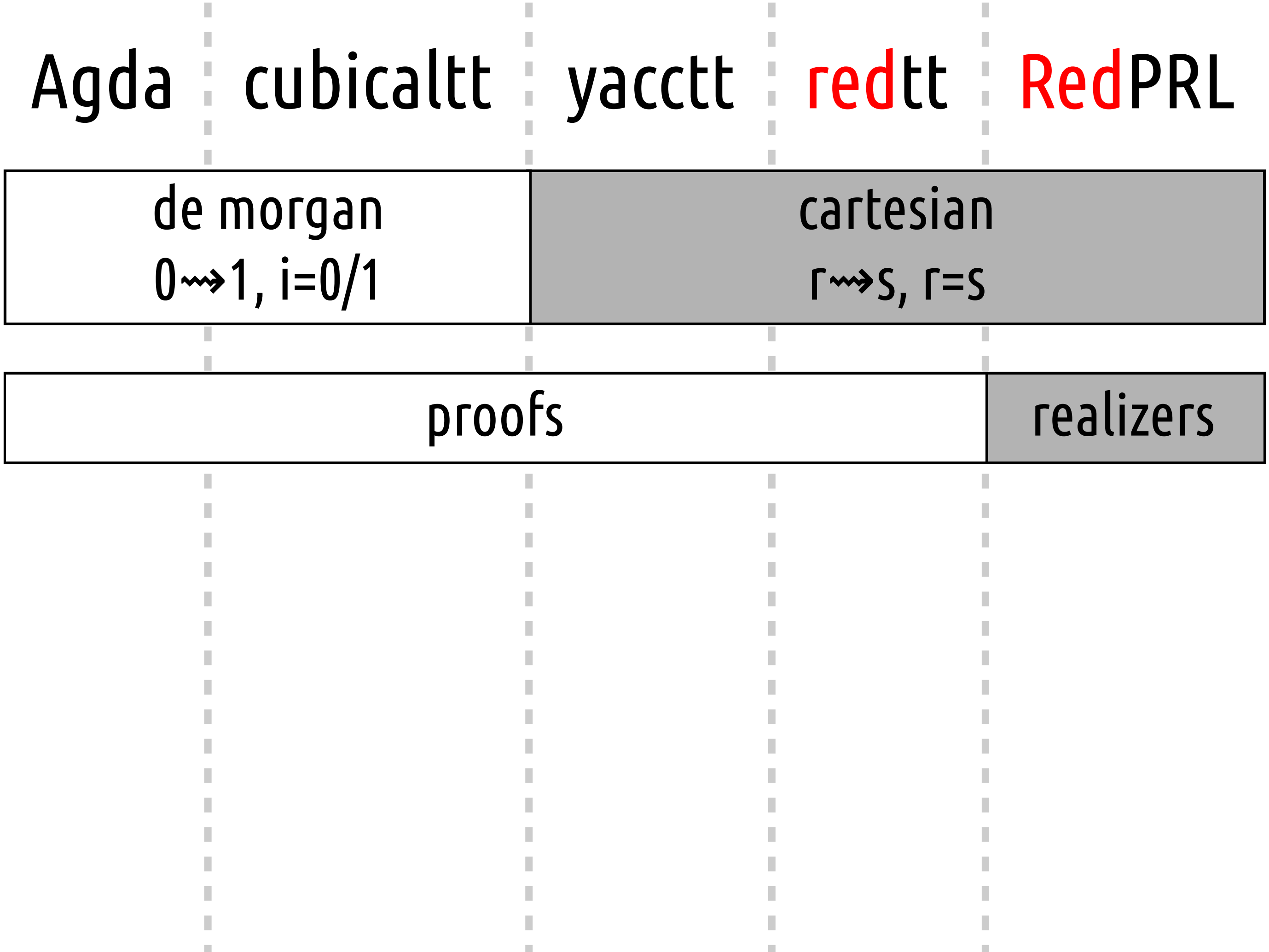
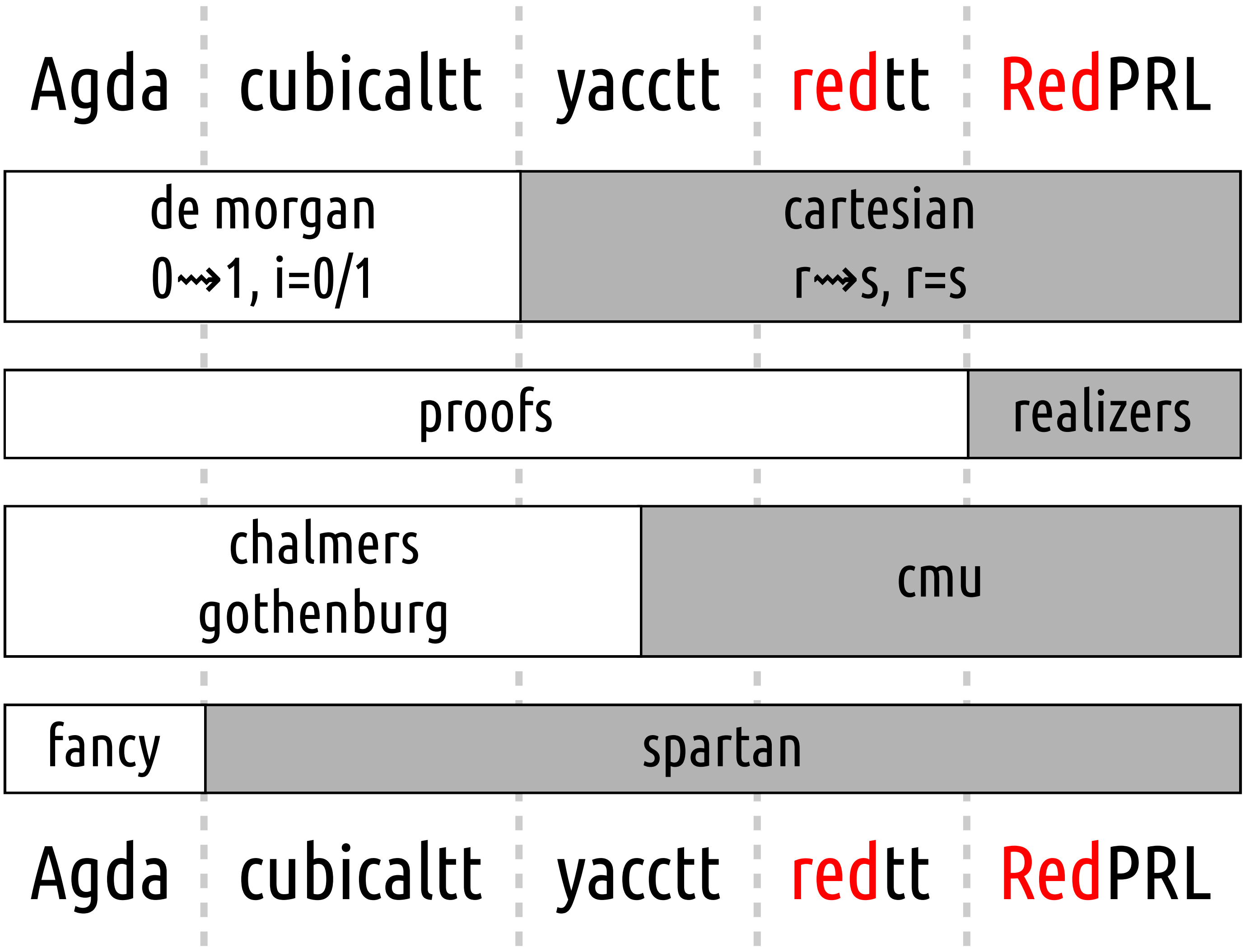
mythos

proofs or realizers?


Agda
cubicaltt
yacctt
redtt
RedPRL
de morgan
0⇝1, i=0/1
cartesian
r⇝s, r=s
proofs
realizers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chalmers
gothenburg
cmu
fancy
spartan
Agda
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redtt
RedPRL

redtt specialities

higher inductive types

two-level type theory

nbe-like algorithm (conjectured correct)

extension types

judgmental refinements

holes, tactics, unification

redtt specialities

higher inductive types

two-level type theory

nbe-like algorithm (conjectured correct)

extension types

judgmental refinements

holes, tactics, unification

see demo

redtt specialities

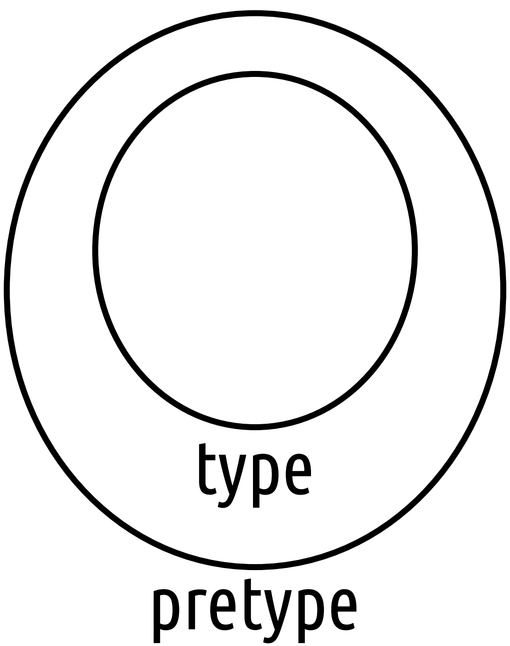
higher inductive types

a general schema; indexed ones on the way

see chtt part 4 [Cavallo & Harper]

redtt specialities

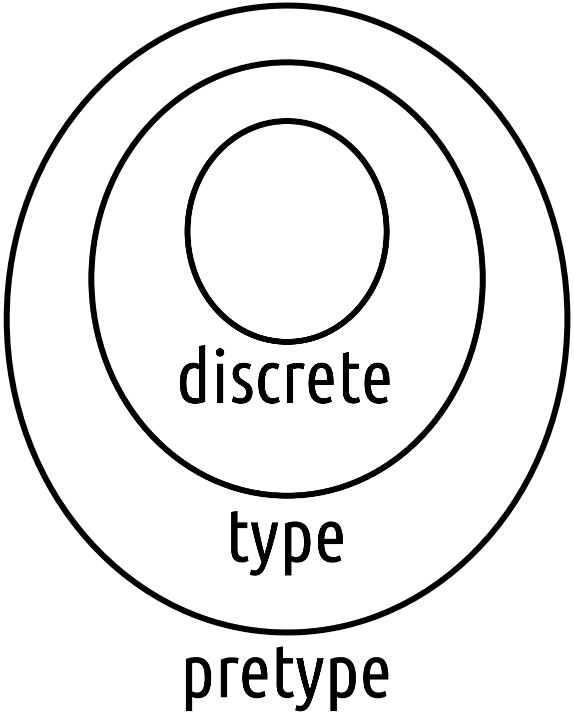
two-level type theory



(no equality types yet)

redtt specialities

todo: many-level type theory



discrete: paths equal to equality

consistent with (strict) UIP

redtt specialities

nbe algorithm

cubicaltt adopts a similar one

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difficulty 1: value re-evaluation: $\mathrm{loop}_x[0/x]$

difficulty 2: constraints: $r=s$

redtt specialities

nbe algorithm

cubicaltt adopts a similar one

difficulty 1: value re-evaluation: $loop_x[0/x]$

difficulty 2: constraints: $r=s$

decidable: $\Phi \vDash r = s$

todo

correctness of nbe

todo

correctness of nbe

equality types

todo

- correctness of nbe
- equality types
- user-defined tactic, pattern matching, etc

todo

correctness of nbe

equality types

user-defined tactic, pattern matching, etc

improved kan operations of universes

todo

correctness of nbe

equality types

user-defined tactic, pattern matching, etc

improved kan operations of universes

synthetic homotopy theory (!)

synthetic homotopy theory