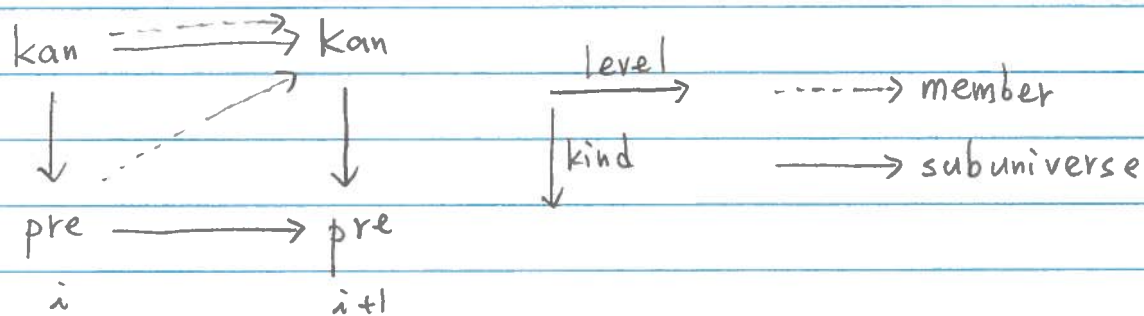


Multiverses

↳ Different kinds of universes

ex: HTS [Voevodsky]

ex: 2-level type theory [Altenkirch]



$A \supseteq$
$A \rightarrow B \rightarrow C \Rightarrow A \rightarrow C$
$A \multimap B \rightarrow C \Rightarrow A \multimap C$

- Kind algebra: subkinding forms a meet-semilattice

$$A \in \mathcal{U}_{l_0}^{k_0}, A \in \mathcal{U}_{l_1}^{k_1}, \dots, A \in \mathcal{U}_{l_n}^{k_n} \Rightarrow A \in \mathcal{U}_{\hat{l}}^{\hat{k}}$$

at least when $\bigwedge_i k_i \leq \hat{k}$ and $\min_i(l_i) \leq \hat{l}$

Better in practice if they form a ~~lattice with a top~~
Heyting semilattice

- Some kinds:

* hcom: having homogeneous comp.

* coe: having coercion

* regular: constant hcom/coe \Rightarrow identity

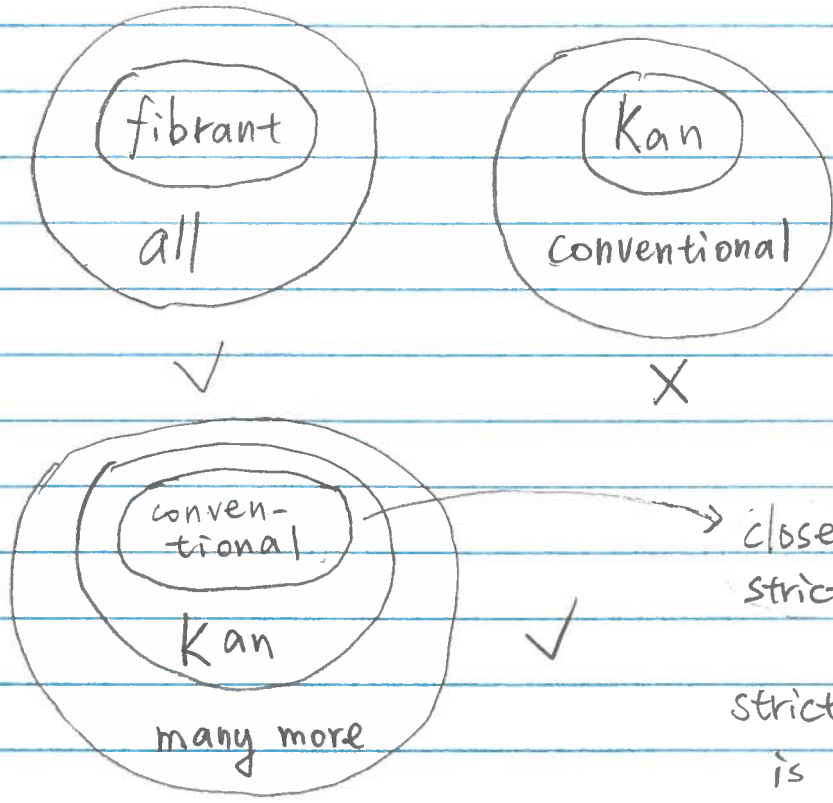
* discrete: kan & all paths trivial

* hprop: resizing

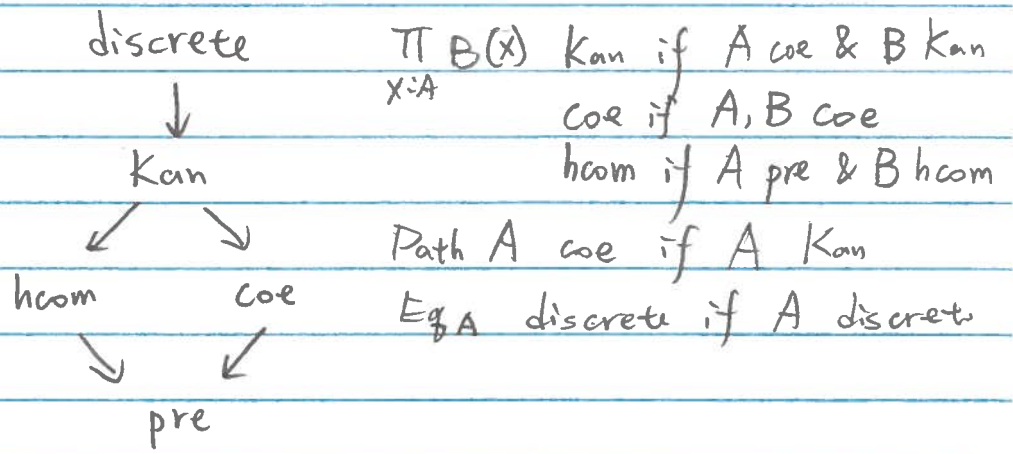
* ...

application

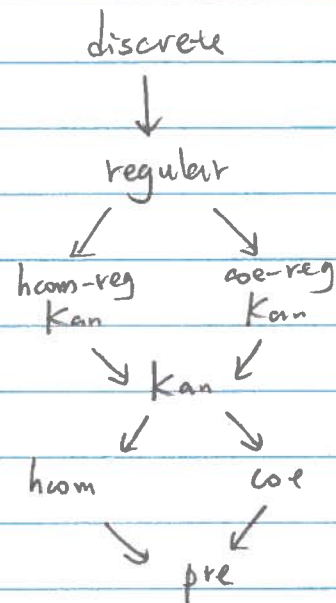
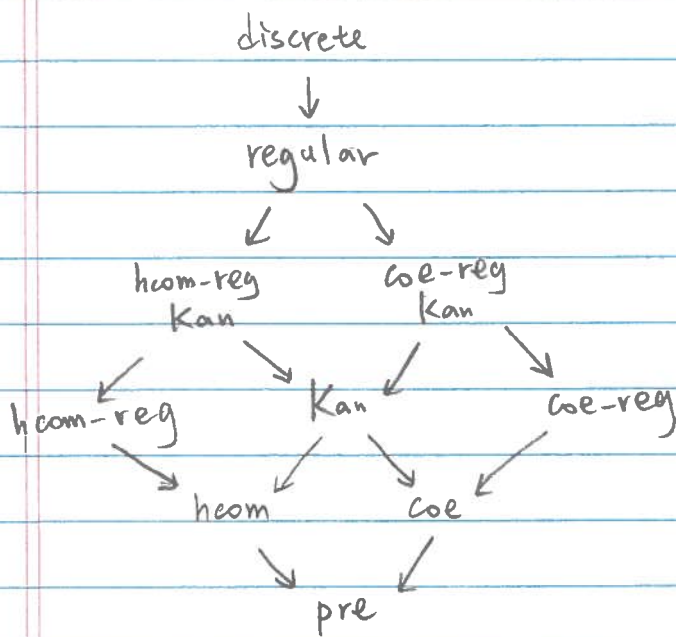
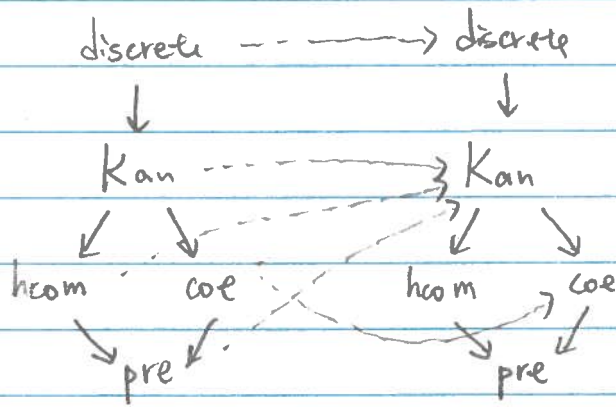
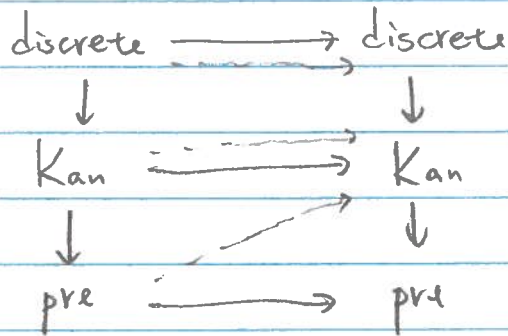
- Discrete: \mathbb{N} , \mathbb{B} , $A \rightarrow B$ if A & B discrete...
- can embed conventional types



strict equalities in \mathbb{N} is still Kan!



Universes



Inductive types

$$\begin{array}{ccc}
 C & \longrightarrow & B \\
 \downarrow & \nearrow \text{glue} & \downarrow \text{right} \\
 A & \xrightarrow{\text{left}} &
 \end{array}
 \quad \text{Kan if } A, B, C \text{ coe}$$

$$\text{coe}(\text{left}(a)) \mapsto \text{left}(\text{coe}(a))$$

$$\text{coe}(\text{right}(b)) \mapsto \text{right}(\text{coe}(b))$$

$$\text{coe}(\text{glue}_x(c)) \mapsto \text{glue}_x(\text{coe}(c))$$

$$\left\{ \begin{array}{l} \text{coe}(\text{glue}_0(c)) \doteq \text{coe}(\text{left}(f(c))) \doteq \text{left}(\text{coe}(f(c))) \\ \text{glue}_0(\text{coe}(c)) \doteq \text{left}(f(\text{coe}(c))) \end{array} \right. \mapsto \text{merid}_0(\text{coe}(c))$$

$$\text{But, for suspension } \text{coe}(\text{merid}_x(c)) \mapsto \text{merid}_x(\text{coe}(c))$$

A different pushout for A, B discrete:

$$\begin{array}{ccc}
 C & \longrightarrow & B \\
 \downarrow & & \downarrow \\
 A & \longrightarrow &
 \end{array}
 \quad \begin{array}{l}
 \text{coe}(\text{glue}^*(c)) \mapsto \text{glue}^*(\text{coe}(c)) \\
 \text{suspension: } A, B := \text{Unit } \underline{\text{discrete}}
 \end{array}$$

— general framework to have $\gg 2$ kinds of universes.

as in