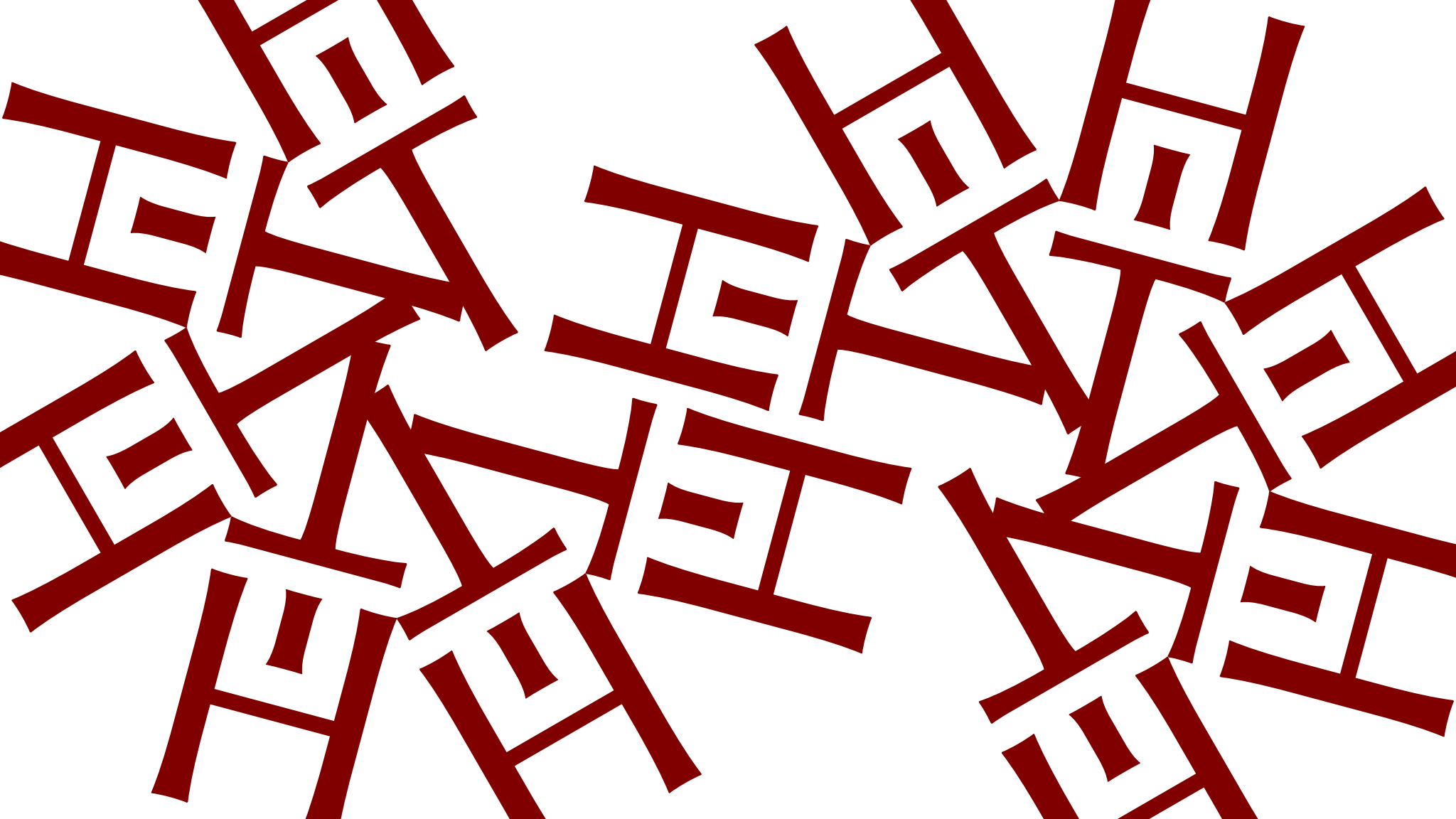
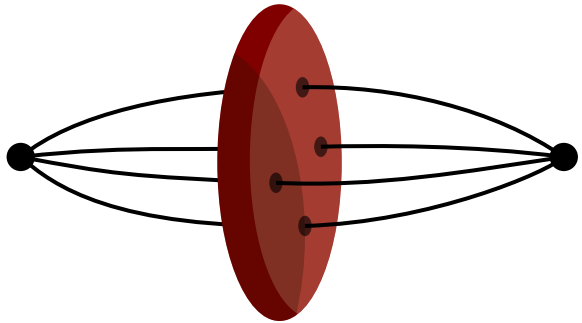
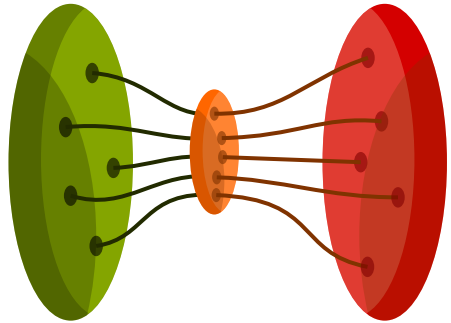
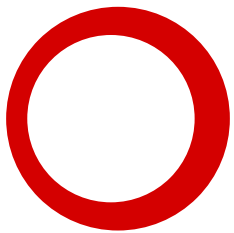




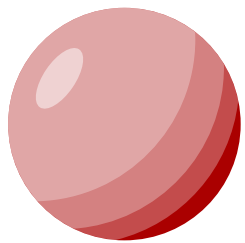
suspension



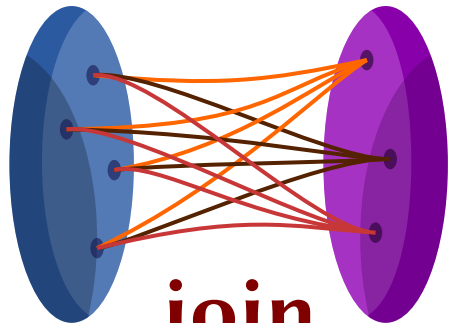
pushout



circle



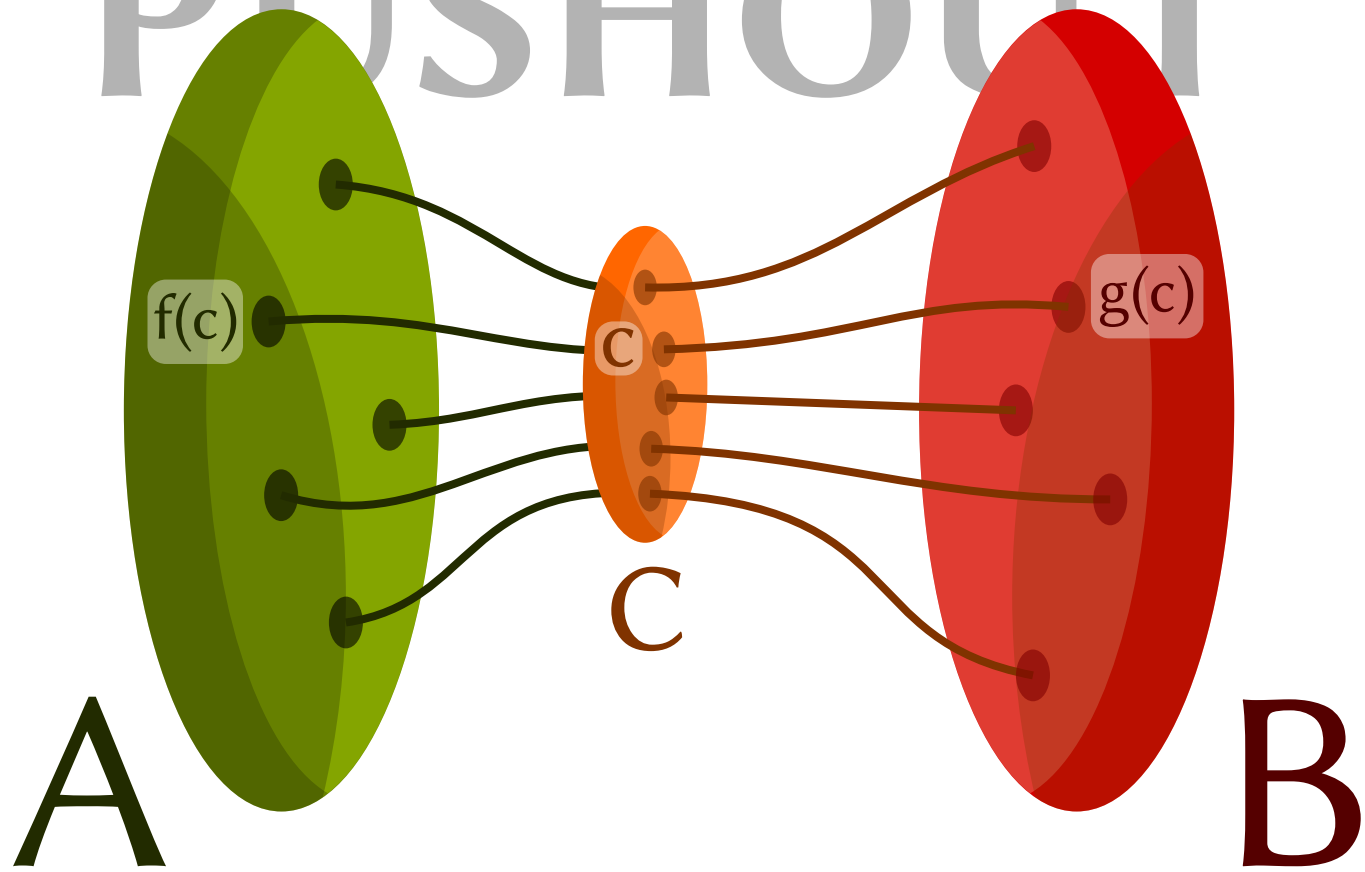
sphere

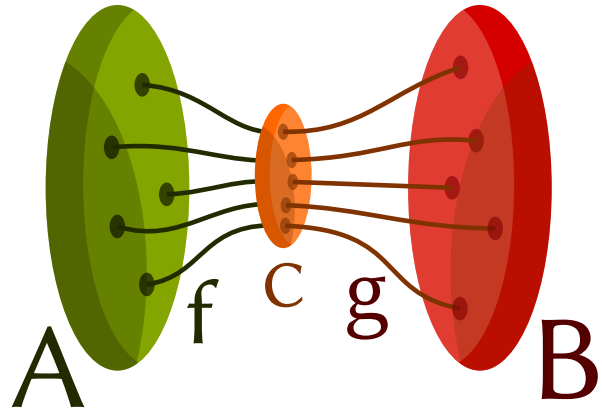


join

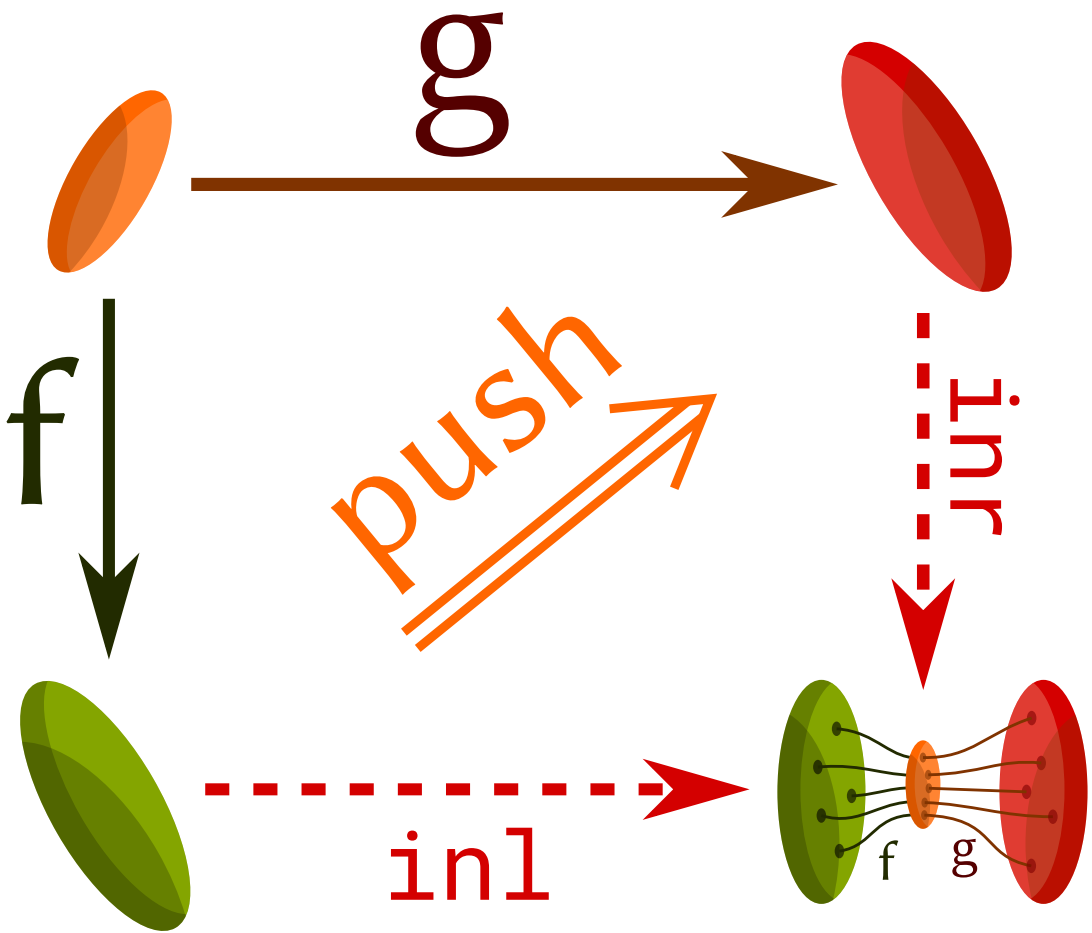
Higher Inductive Types

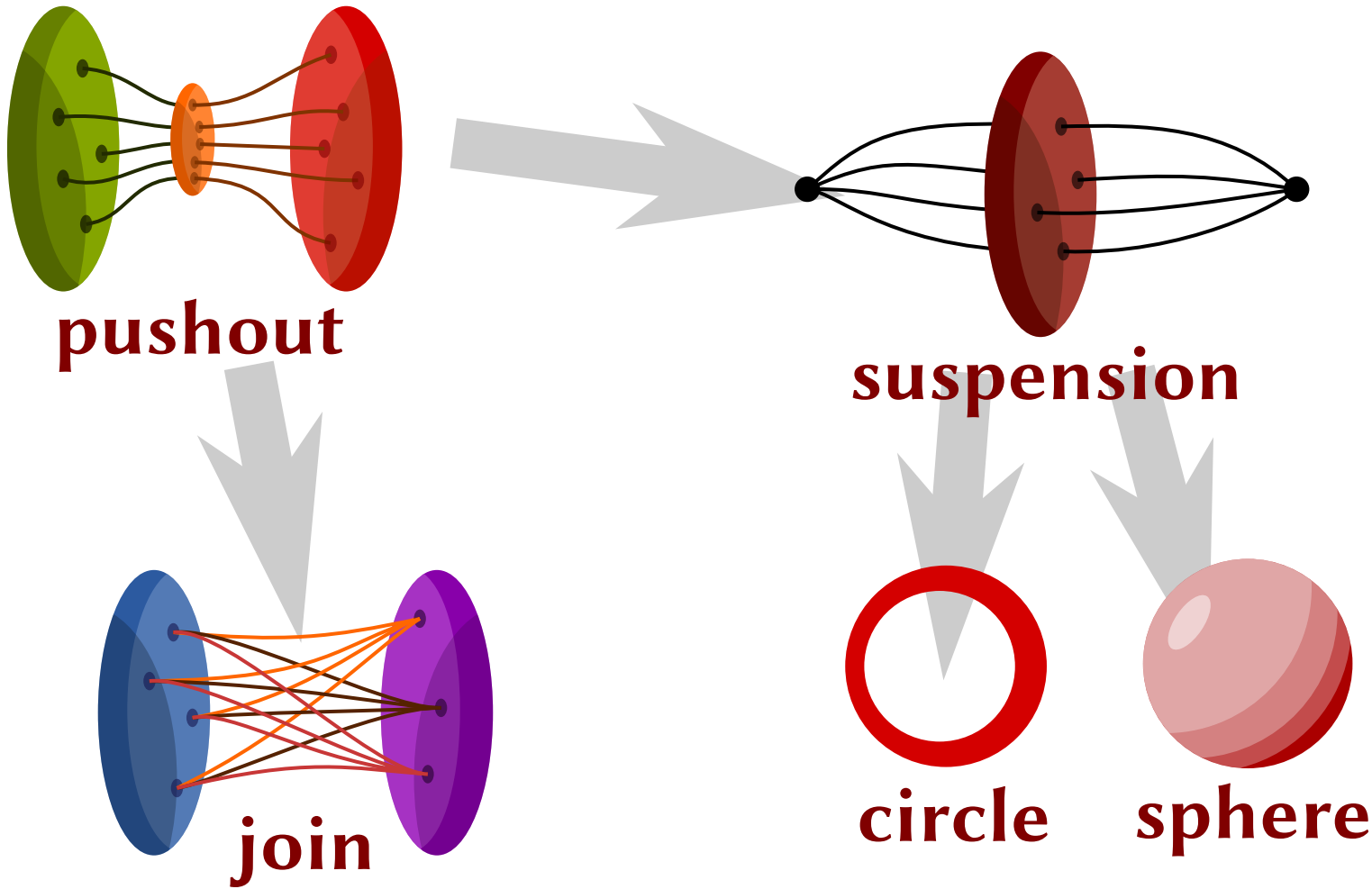
PUSHOUT



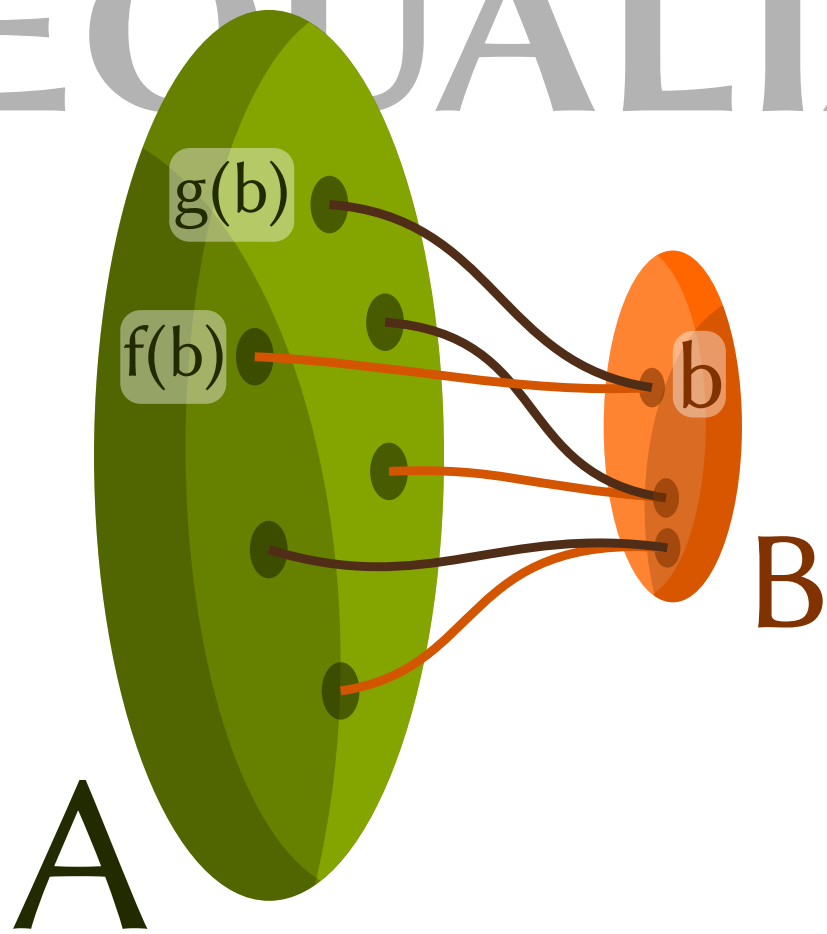


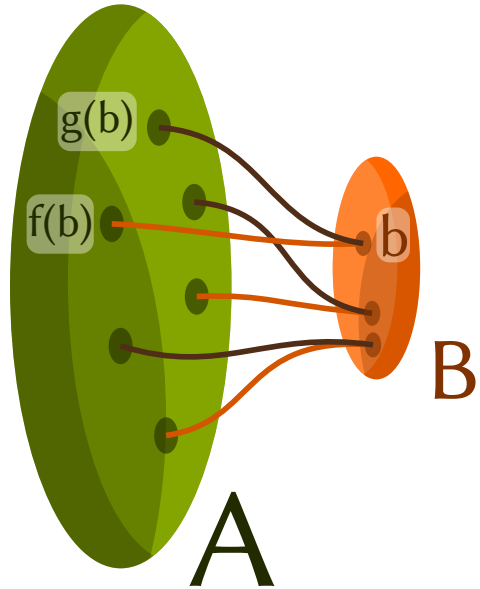

```
data Pushout {A B C : U}
  (f : C → A) (g : C → B) : U where
  inl : A → Pushout f g
  inr : B → Pushout f g
  push : (c : C) → inl (f c) ≡ inr (g c)
```


g
f
push
inr
inl
f
g

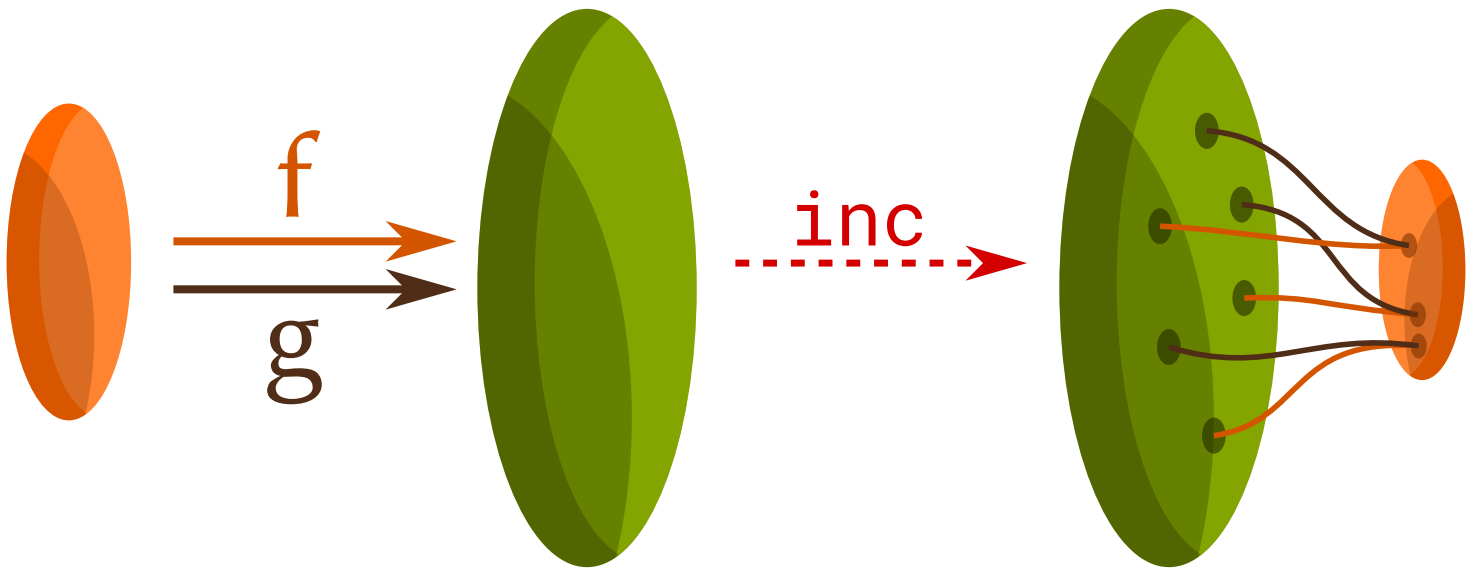

pushout
suspension
join
circle
sphere

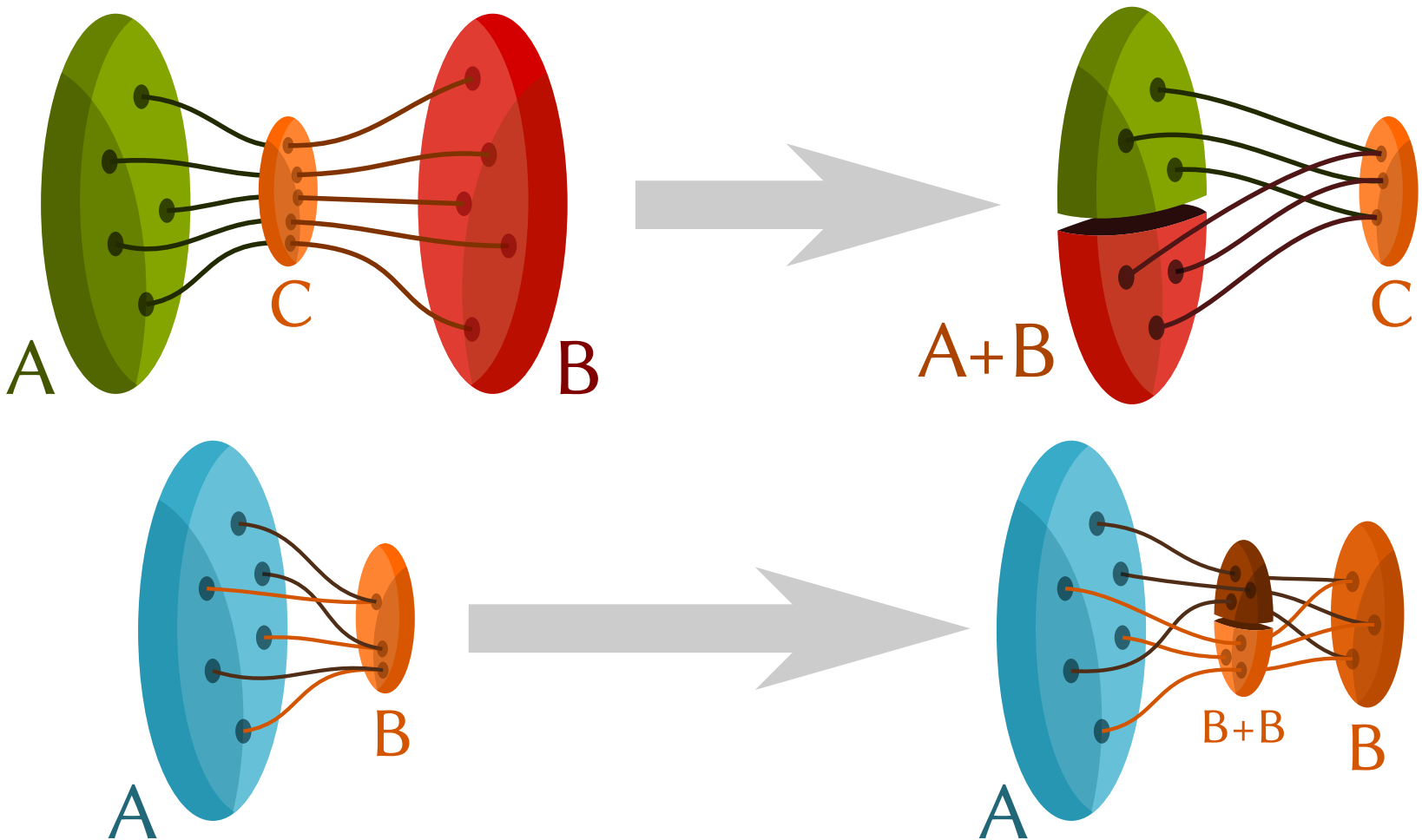
COEQUALIZER



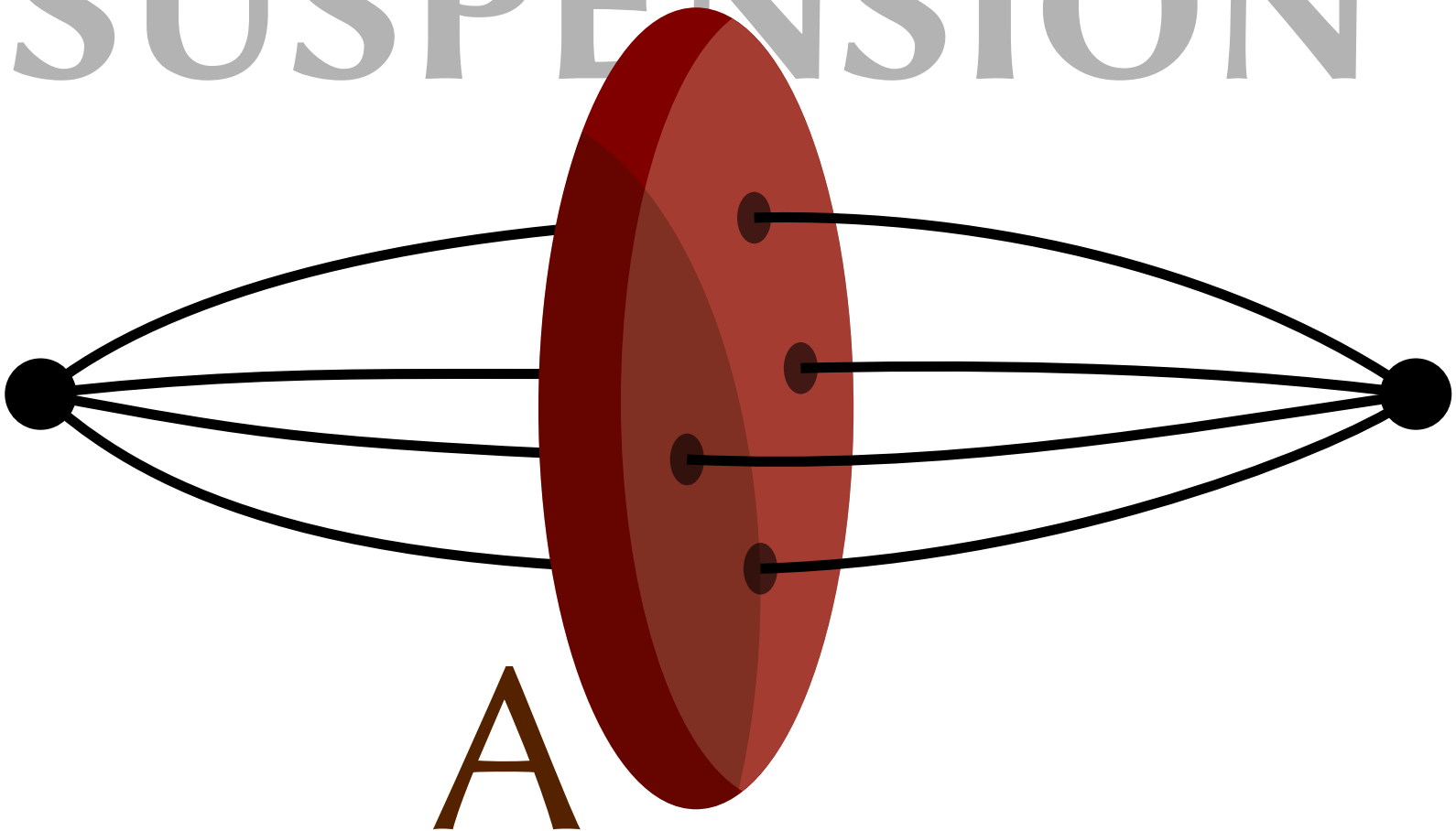



```
data Coequalizer {A B : U}
       (f g : B → A) : U where
  inc : A → Coequalizer f g
  eq : (b : B) → inc (f b) ≡ inc (g b)
```

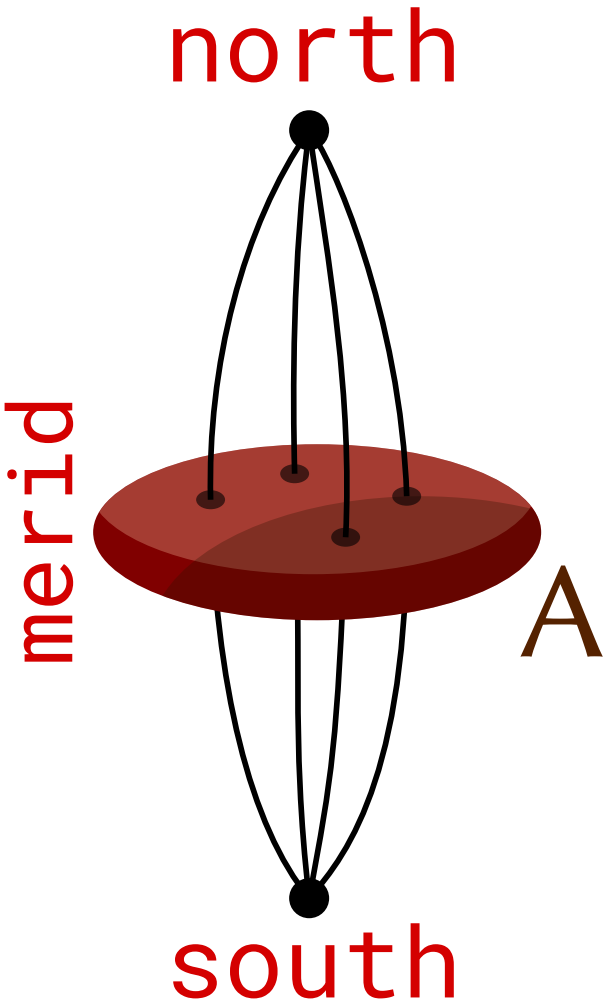

f
g
inc


A
C
B
A+B
C
A
B
B+B
B

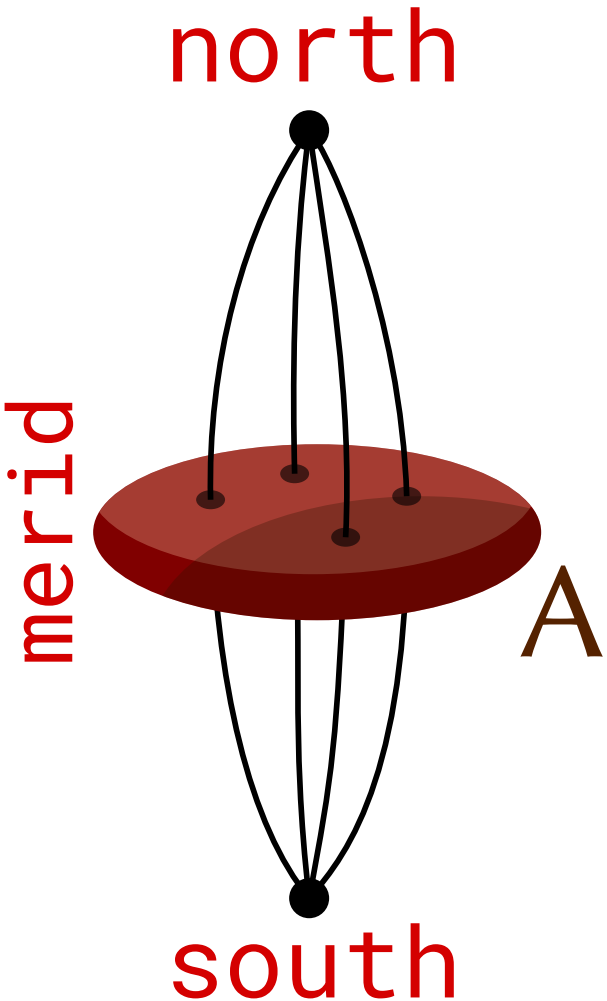
SUSPENSION

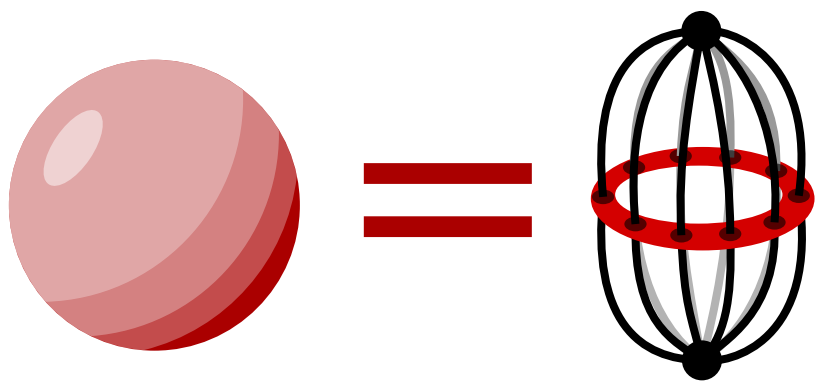


A




```
data Susp (A : U) : U where
  north : Susp A
  south : Susp A
  merid : (a : A) → north ≡ south
```

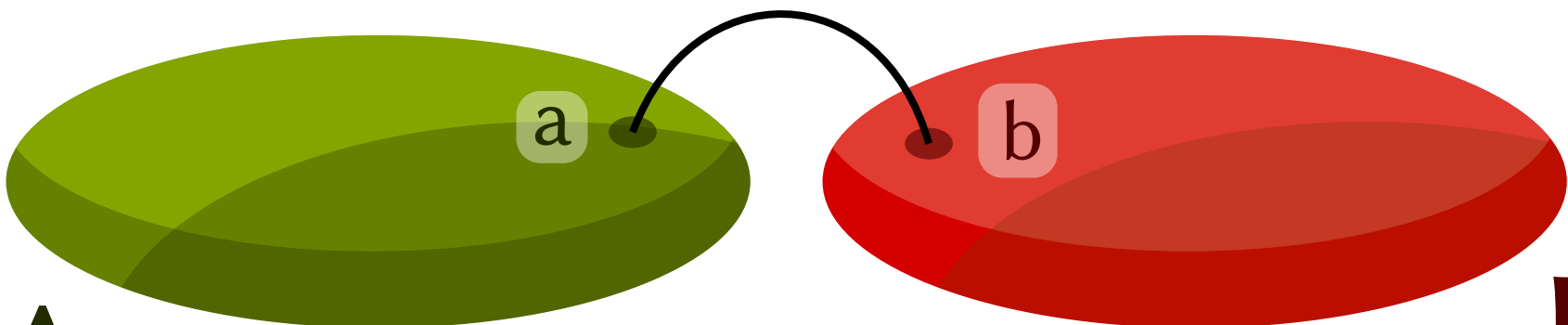




$S^n := \mathrm{Susp}^n(2)$

See the lecture on truncation levels

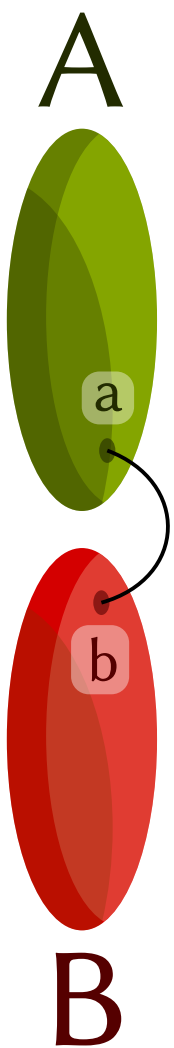
WEDGE



A

B

sum types for pointed types



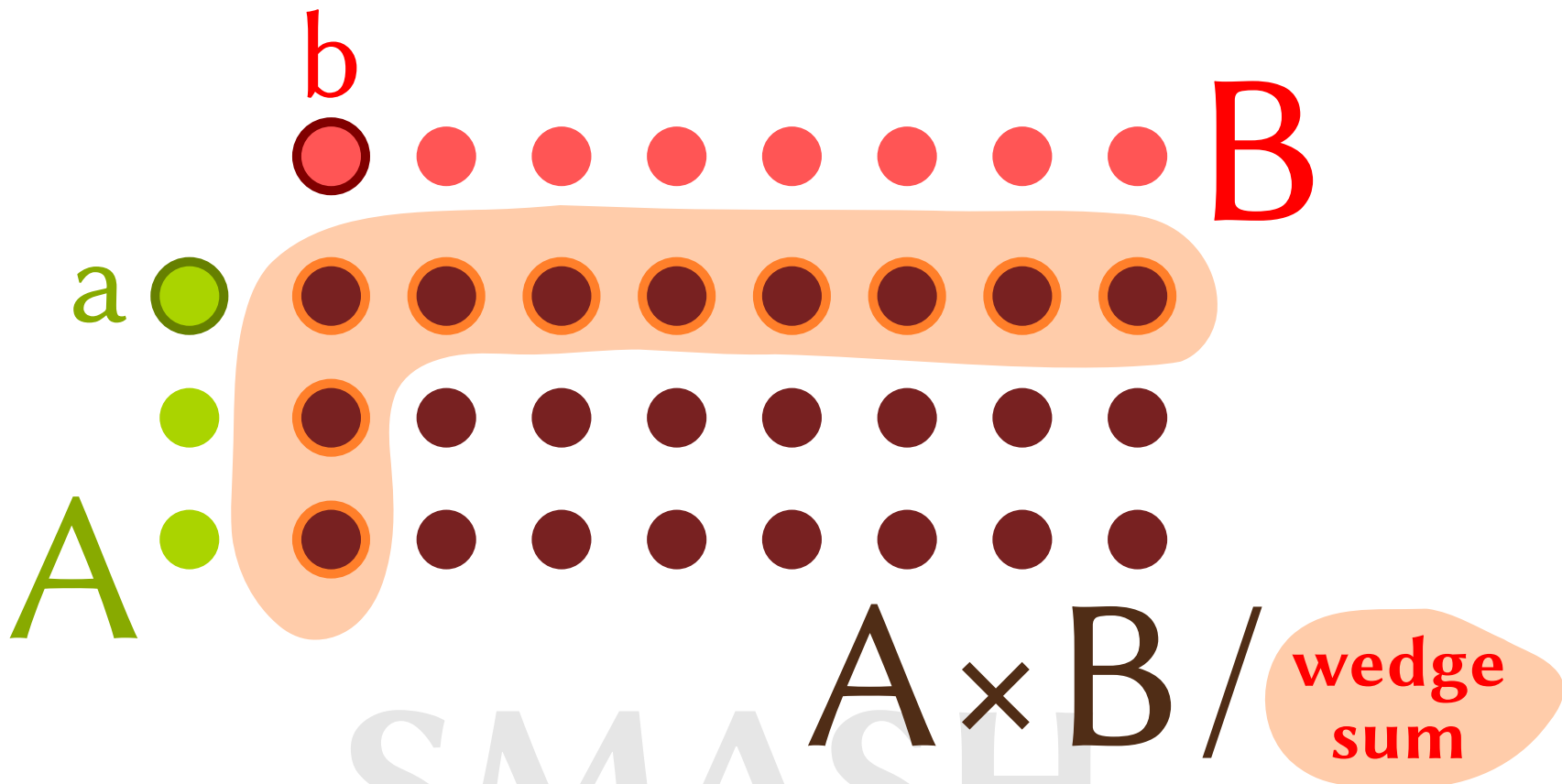

```
data Wedge (A B : U)
  (a : A) (b : B) : U where

  inl : A → Wedge A B a b
  inr : A → Wedge A B a b
  glue : inl a ≡ inr b
```

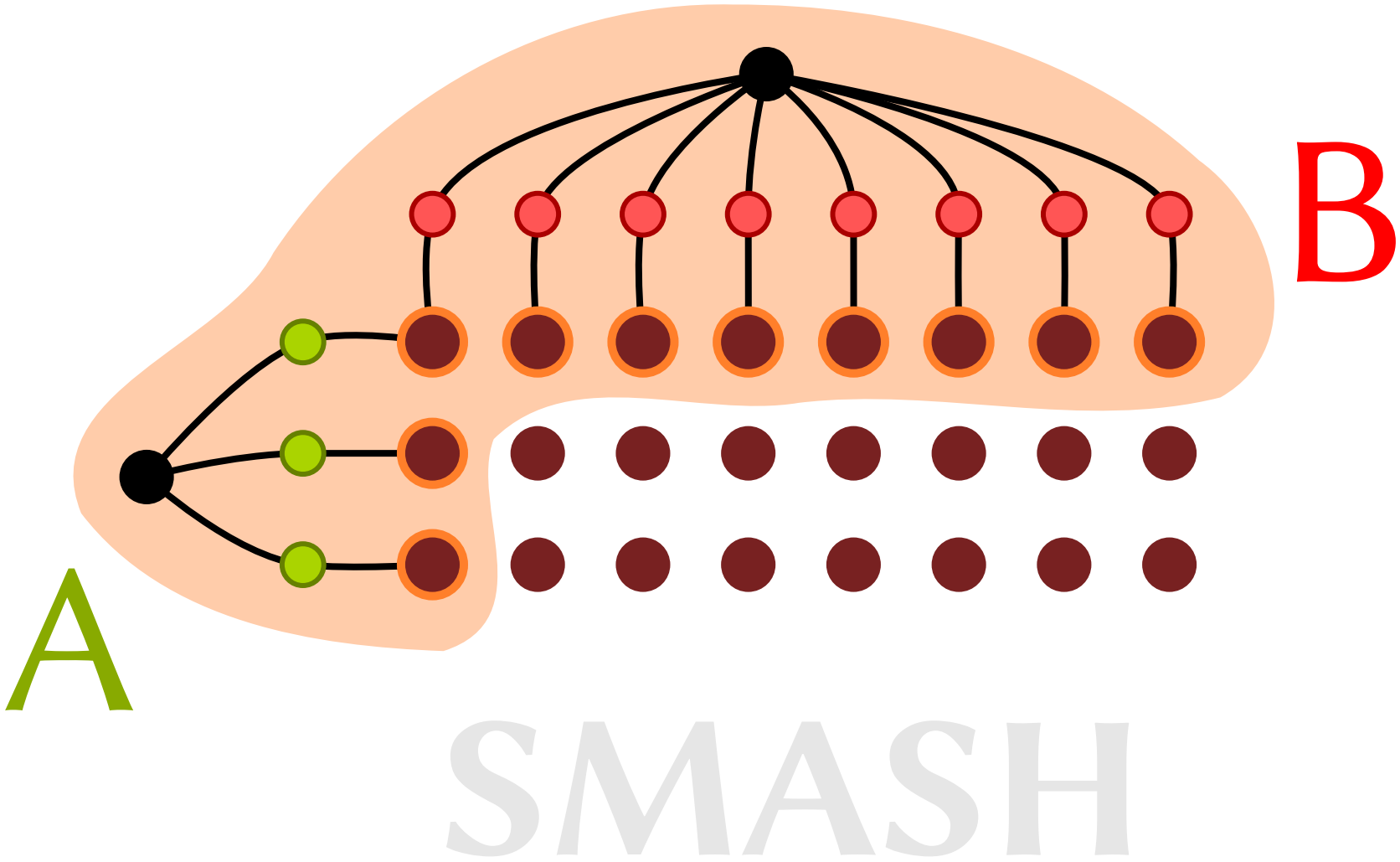
SMASH

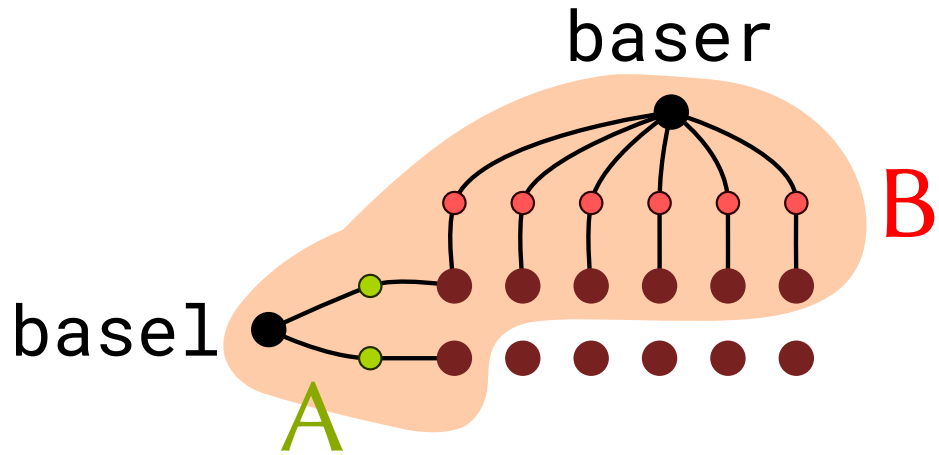
$$A \wedge B := A \times B \,/\, A \vee B$$

smash

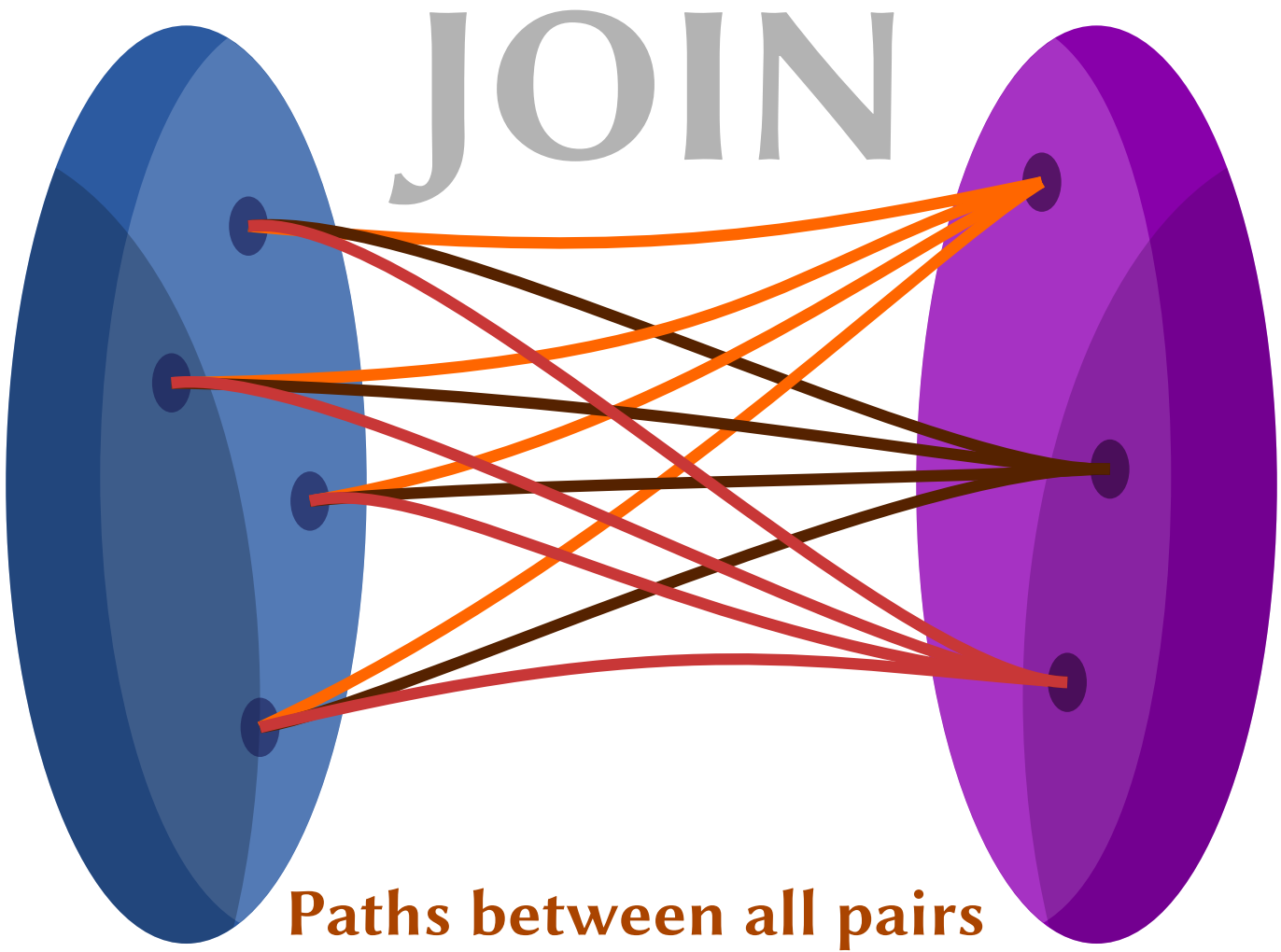
wedge sum

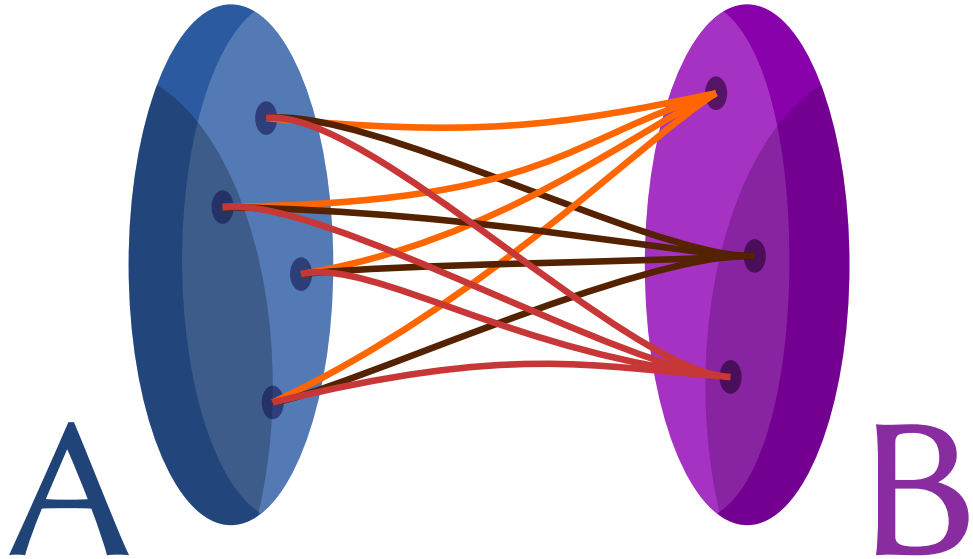

b
B
a
A
A×B/
wedge
sum
SMASH


B
A
SMASH




```
data Smash (A B : U) (a : A) (b : B) : U where
  pair : A → B → Smash A B a b
  basel : Smash A B a b
  baser : Smash A B a b
  gluel : (a' : A) → inc a' b ≡ basel
  gluer : (b' : B) → inc a b' ≡ baser
```


JOIN
Paths between all pairs




```
data Join (A B : U) : U where
  inl : A → Join A B
  inr : B → Join A B
  join : (a : A) (b : B) → inl a ≡ inr b
```

$$X \star Y \simeq \mathrm{Susp}\,(X \wedge Y)$$

join smash

X, Y and Z are pointed types

$$\mathrm{Susp}\,(X \wedge Y) \simeq (\mathrm{Susp}\,X) \wedge Y \simeq X \wedge (\mathrm{Susp}\,Y)$$

$$S^n \wedge S^m \simeq S^{n+m}$$

$$S^n \star S^m \simeq S^{n+m+1}$$

$$A \times B \longrightarrow C \simeq A \longrightarrow (B \longrightarrow C)$$

$$X \wedge Y \cdot\!\!\longrightarrow Z \simeq X \cdot\!\!\longrightarrow (Y \cdot\!\!\longrightarrow Z)$$

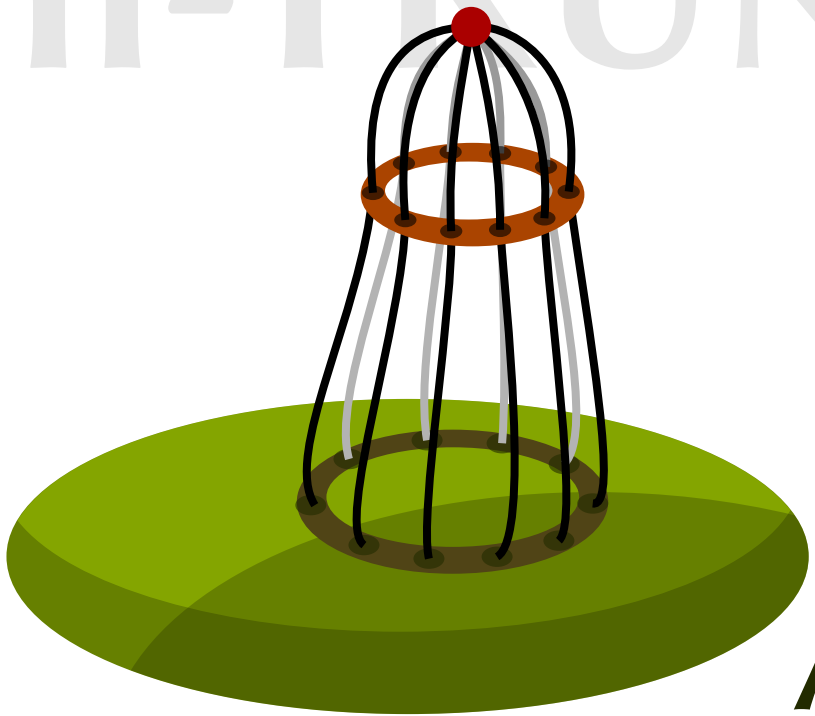
point-preserving functons

CURRYING

n-TRUNCATION

Best n-type approximation

n-TRUNCATION

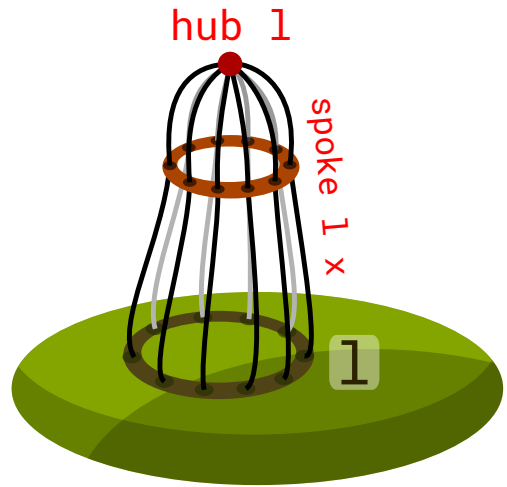


Fill every image of S^{n+1} with a cone

See the lecture on truncation levels

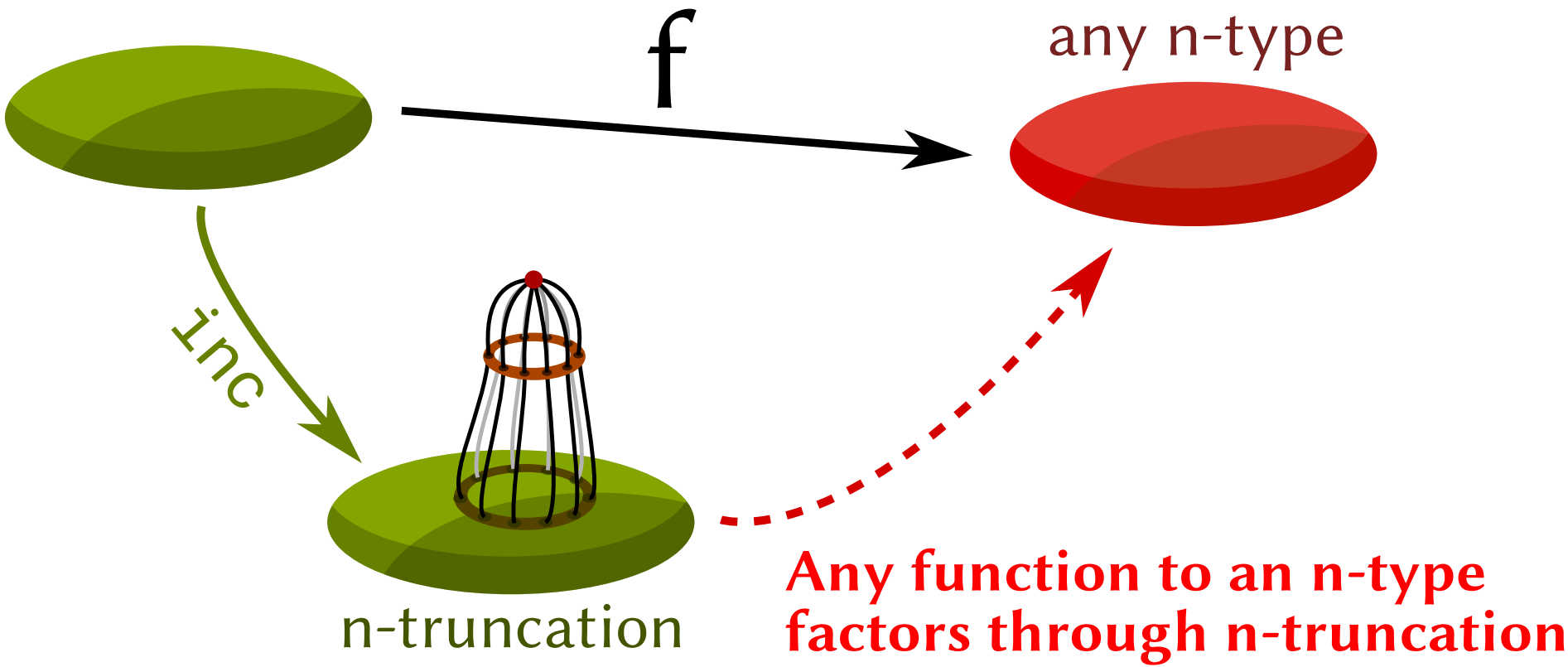
also [HoTT, 7.3]

A




```
data Trunc n (A : U) : U where
  inc : A → Trunc n A
  hub   : (S^(n+1) → A) → Trunc n A
  spoke : (l : S^(n+1) → A) (x : S^(n+1)) → hub f ≡ f x
```

effectively **has-level n (Trunc n A)** [HoTT, 7.3]


f
any n-type
inc
n-truncation
Any function to an n-type
factors through n-truncation

More: set quotients

coequalizer + 0-truncation

More^2: sequential colimits

e.g. define S^∞ as $\varinjlim S^n$

More^3: textbooks or ask Favonia

All definable using pushouts