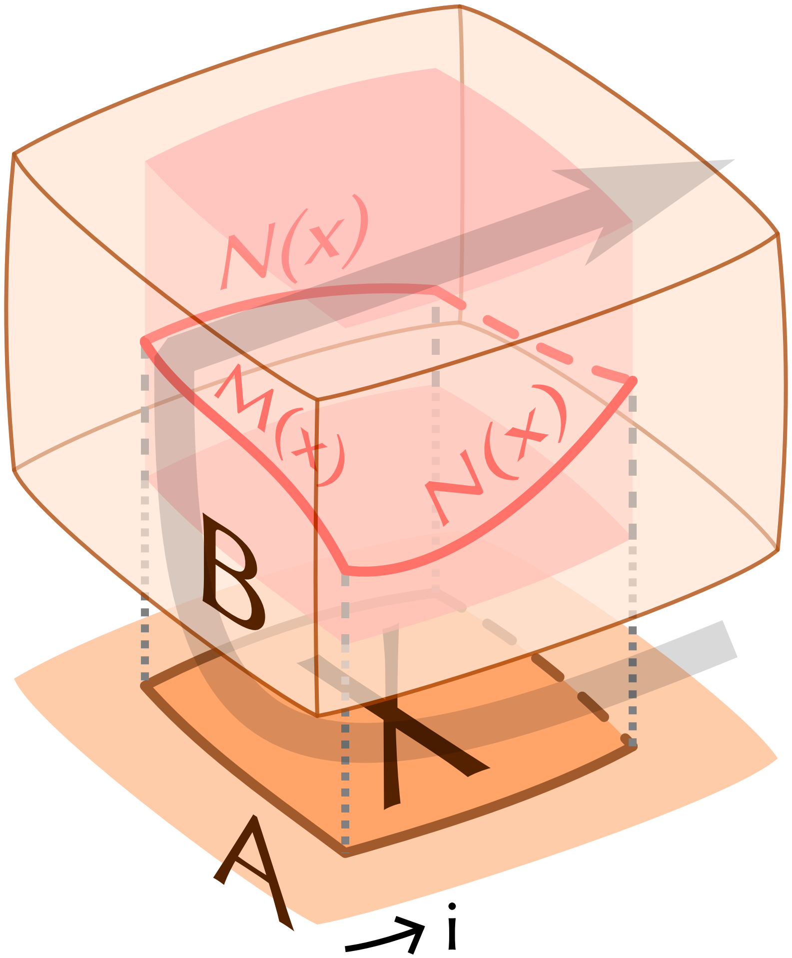
K ∀ И

Correction:

We will redo the empty type

the rules were wrong; the video was re-uploaded

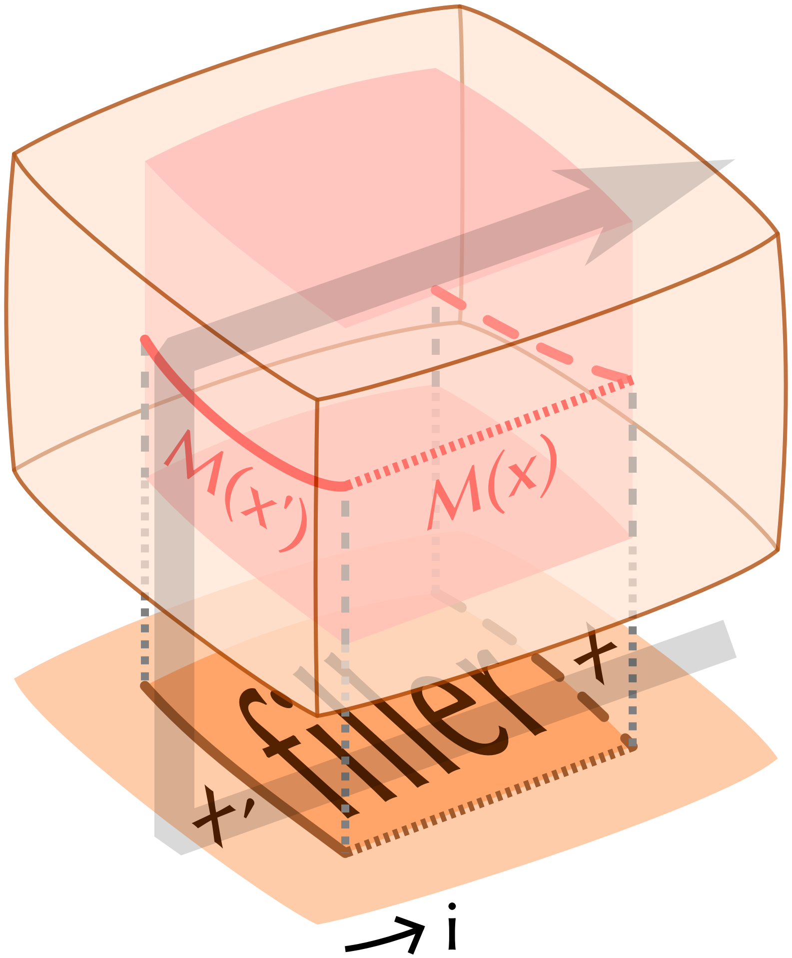


$$\mathrm{hcomp}^i\ (\Pi_{x:A} B)\ [\varphi \mapsto N]\ M$$

$$\rightsquigarrow$$

$$\lambda x.\mathrm{hcomp}^i\ B\ [\varphi \mapsto N(x)]\ M(x)$$

fill the square with the same x along i



$\mathsf{transp}^i\ (\Pi_{x:A} B)\ \varphi\ M$

$\downarrow$

$\lambda x.\mathsf{transp}^i\ B[\mathsf{filler}^i(x)/x]\ \varphi\ M(\mathsf{filler}^0(x))$

$\mathsf{filler}^i(x) :\equiv \mathsf{transp\text{-}fill}^{\sim i}\ A\ \varphi\ x$

back and forth

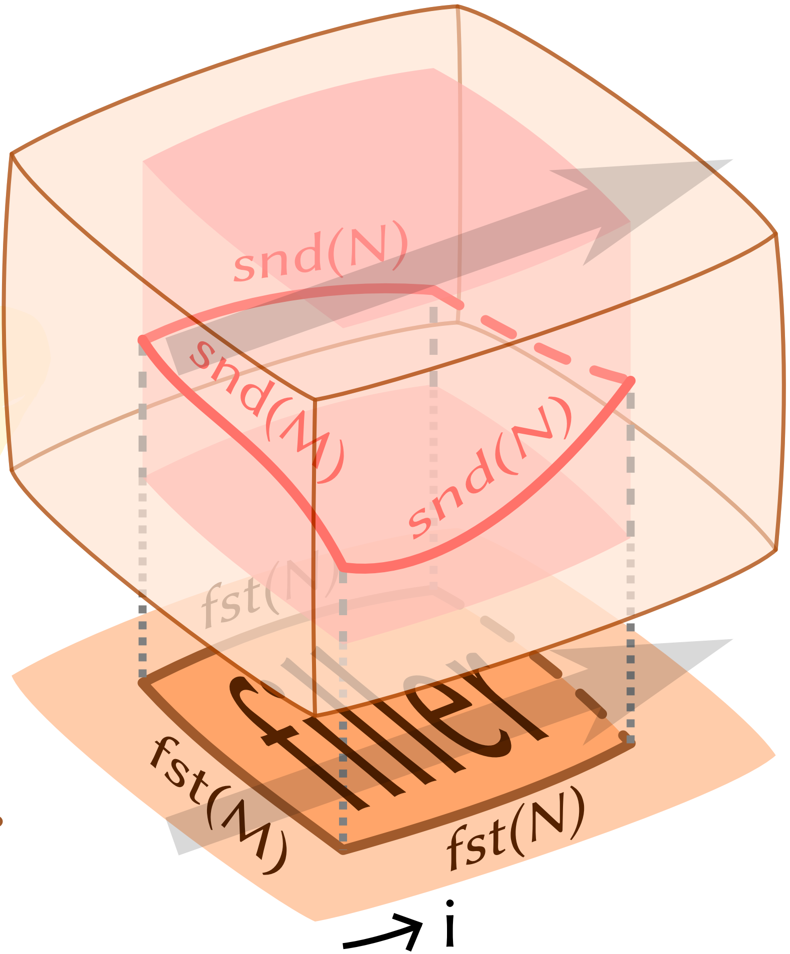
$$\mathrm{hcomp}^i\ (\Sigma_{x:A} B)\ [\varphi \mapsto N]\ M$$

$$\downarrow$$

$$\langle\ \mathrm{hcomp}^i\ A\ [\varphi \mapsto \mathrm{fst}(N)]\ \mathrm{fst}(M)\ ,$$
$$\mathrm{comp}^i\ B[\mathrm{filler}^i/x]\ [\varphi \mapsto \mathrm{snd}(N)]\ \mathrm{snd}(M)\ \rangle$$

$$\mathrm{filler}^i :\equiv \mathrm{hfill}^i\ A\ [\varphi \mapsto \mathrm{fst}(N)]\ \mathrm{fst}(M)$$

needs comp^i because B(filler/x) depends on i



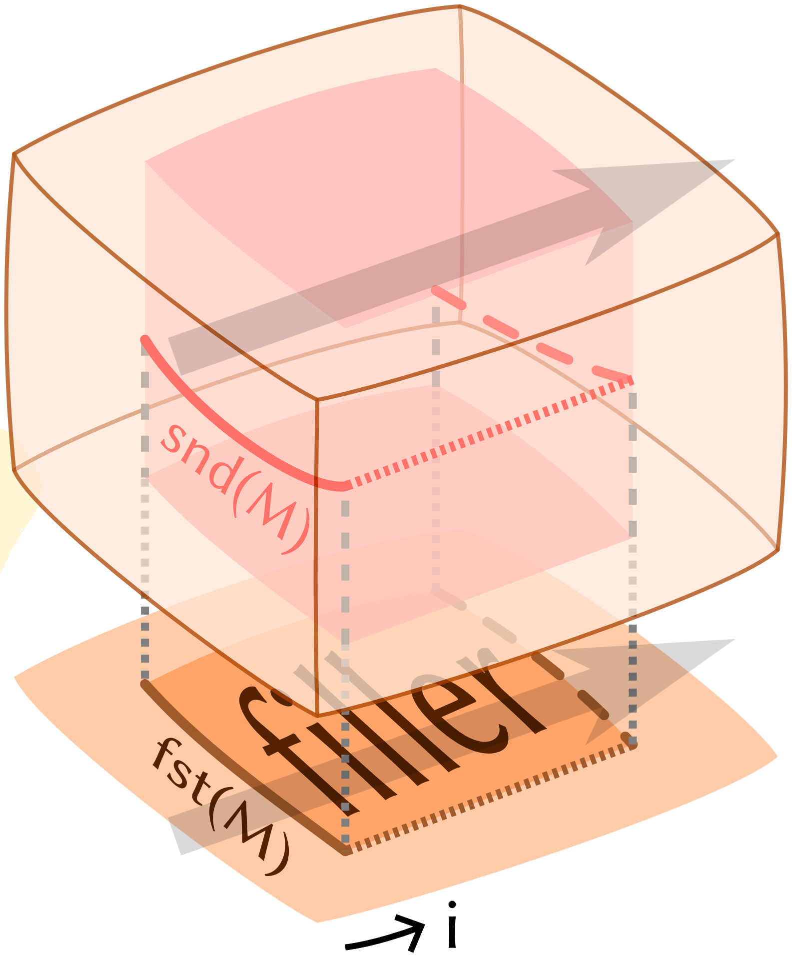
$\mathrm{transp}^i\ (\Sigma_{x:A} B)\ \varphi\ M$

$\downarrow$

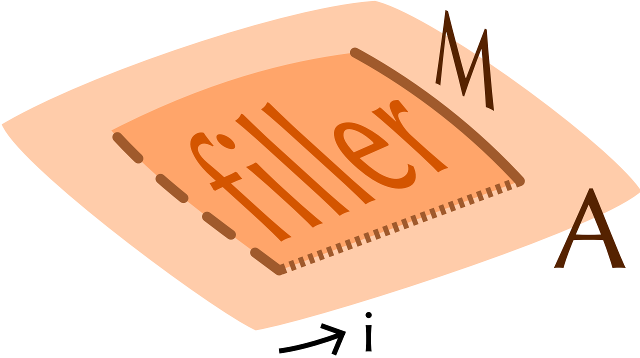
$\langle\ \mathrm{transp}^i\ A\ \varphi\ \mathrm{fst}(M)\ ,$

$\mathrm{transp}^i\ B[\mathrm{filler}^i/x]\ \varphi\ \mathrm{snd}(M)\ \rangle$

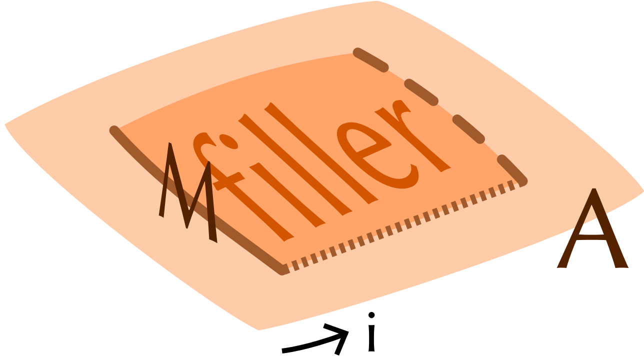
$\mathrm{filler}^i :\equiv \mathrm{transp\text{-}fill}^i\ A\ \varphi\ \mathrm{fst}(M)$



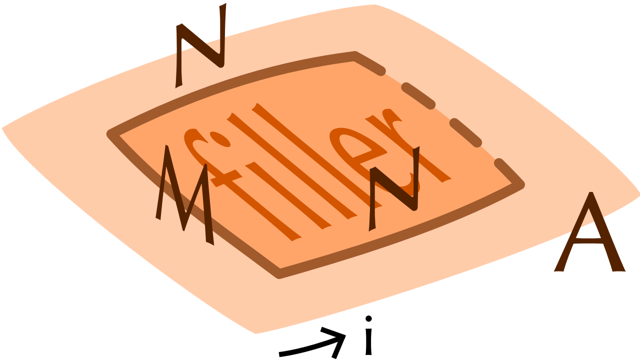
DERIVED OPERATORS



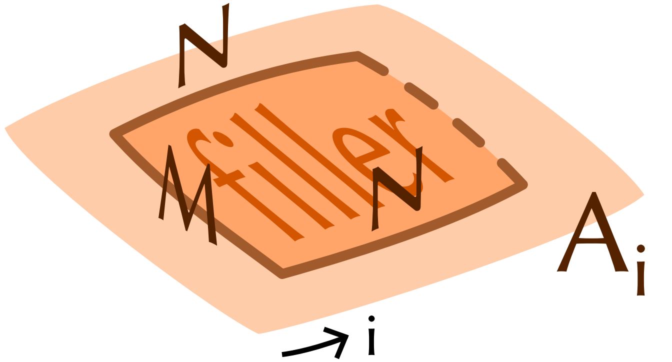
$\text{transp-fill}^{\sim i}\ A\ \varphi\ M$



$\text{transp-fill}^{i}\ A\ \varphi\ M$



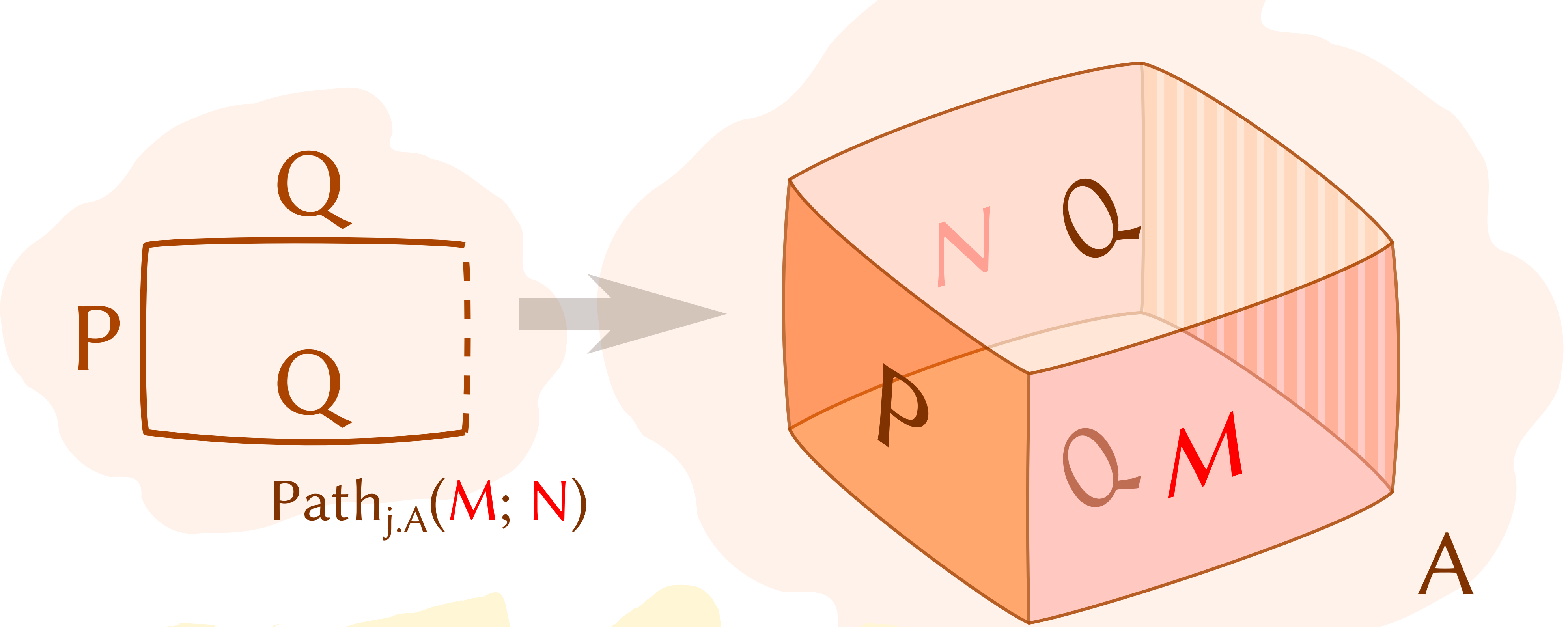
$\text{hfill}^{i}\ A\ [\varphi \mapsto N]\ M$



$\text{comp}^{i}\ A\ [\varphi \mapsto N]\ M$

- ✓ the unit
- ✓ functions
- ✓ pairs
- paths

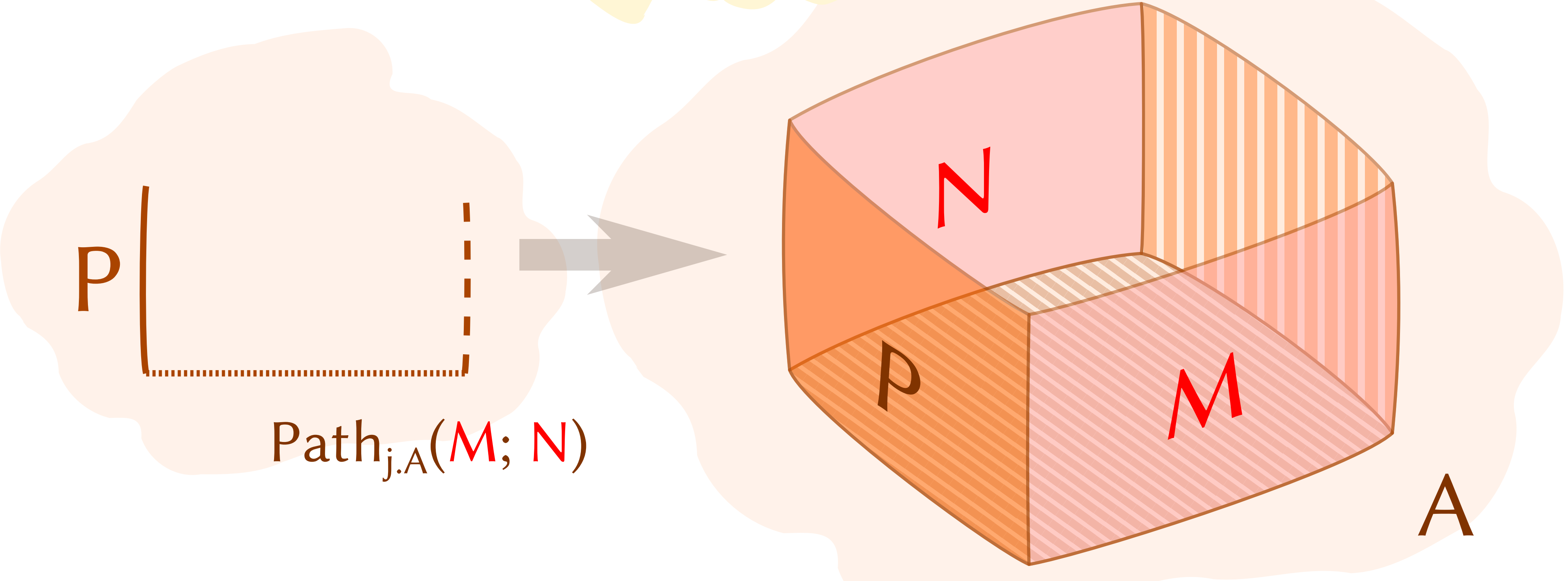
- natural numbers
- disjoint sums
- the empty type
- the circle
- universes



$$\text{hcomp}^i\,(\text{Path}_{j.A}(M; N))\;[\varphi \mapsto Q]\;P$$

$$\equiv \lambda j.\text{hcomp}^i\;A\;[\varphi \mapsto Q@j,\; j{=}0 \mapsto M,\; j{=}1 \mapsto N]\;P@j$$

$$\mathrm{transp}^i\ (\mathrm{Path}_{j.A}(M;\ N))\ \varphi\ P$$
$$\equiv \lambda j.\mathrm{comp}^i\ A\ [\varphi \mapsto P@j,\ j{=}0 \mapsto M,\ j{=}1 \mapsto N]\ P@j$$

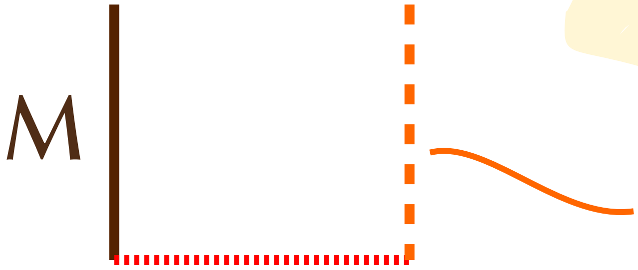


- ✓ the unit
- ✓ functions
- ✓ pairs
- ✓ paths

negative types

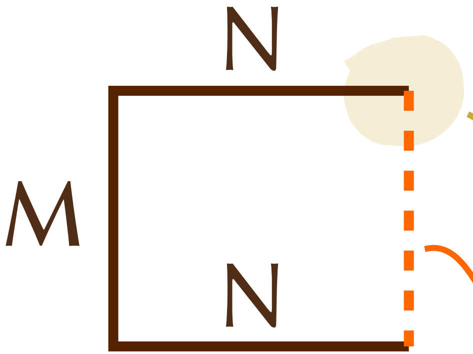
- natural numbers
- disjoint sums
- the empty type
- the circle
- universes

positive types



$\mathrm{transp}^i\ \mathbb{N}\ \varphi\ M$

M always works

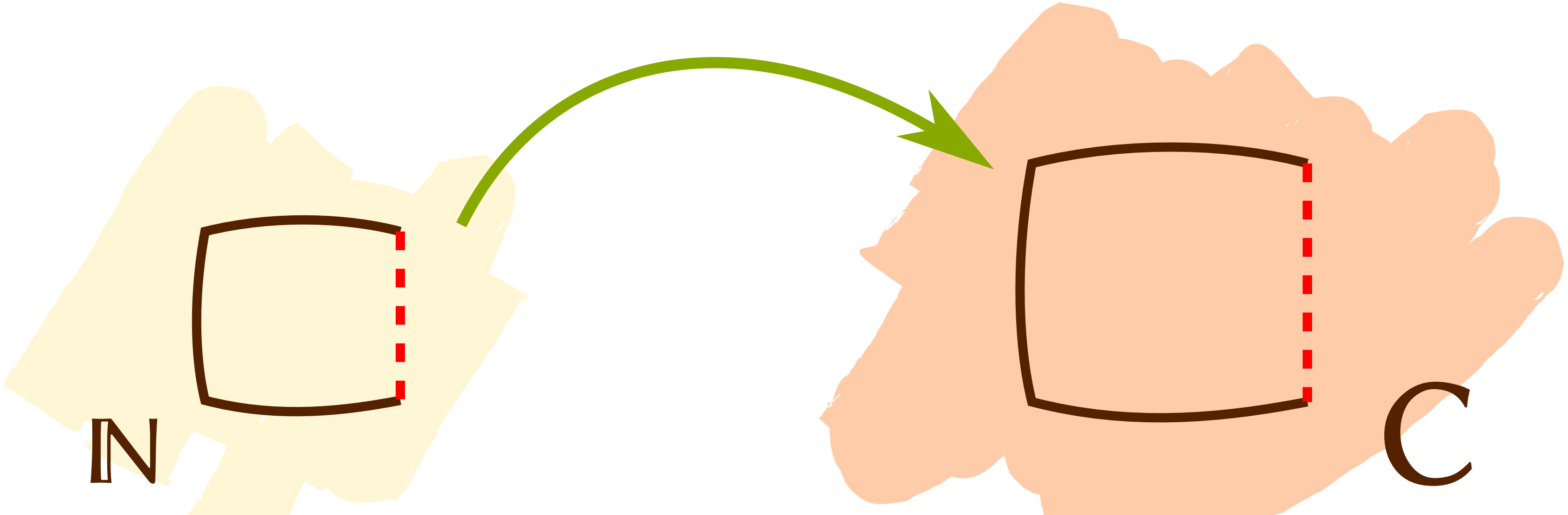


$$\mathrm{hcomp}^i\ \mathbb{N}\ [\varphi \mapsto N]\ M$$

If φ = true, this should reduce to $N[1/i]$

Otherwise, what should we do?

Freely generated inductive types now have irreducible hcomps



elim(hcomp) = comp(elim)

$$E(\mathsf{hcomp}^i\ \mathbb{N}\ [\varphi \mapsto O]\ P) \equiv \mathsf{comp}^i\ C[\mathsf{filler}^i/x]\ [\varphi \mapsto E(O)]\ E(P)$$

$$E(O) :\equiv \mathsf{elim}_{\mathbb{N}}[x.C](M;\ x.y.N;\ O)$$

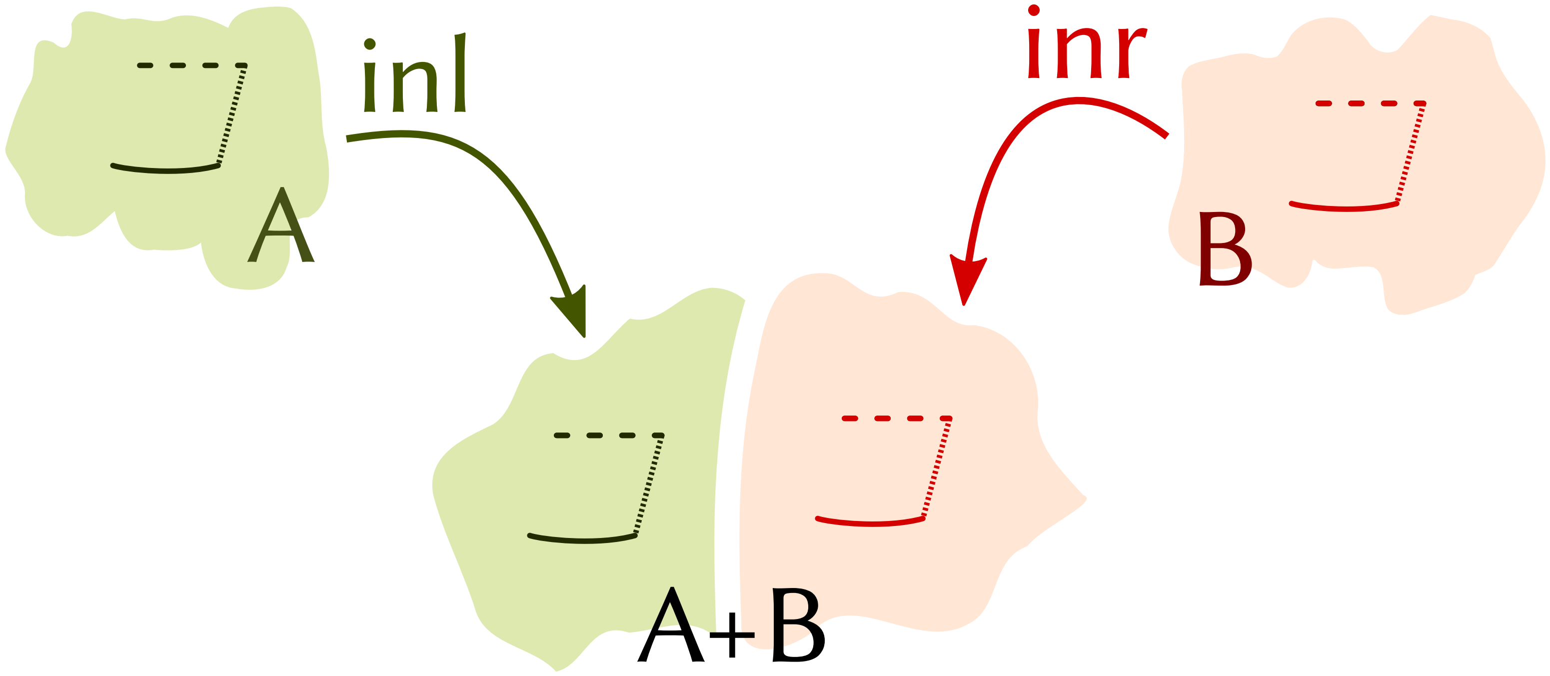
$$\mathsf{filler}^i :\equiv \mathsf{hfill}^i\ \mathbb{N}\ [\varphi \mapsto O]\ P$$

1. Inductive types have formal hcomps
2. Elim commutes with formal hcomps

e.g., $\bot$, $\mathbb{N}$, 2, A+B, the circle, etc.

transp can always be reduced*

**except for indexed inductive families*



transp (A+B) φ inl(M) ≡ inl(transp A φ M)

transp (A+B) φ inr(M) ≡ inr(transp B φ M)

*Optional**

hcomp *can* commute with constructors

ℕ

$\mathrm{suc}(\mathrm{hcomp}^i\ \mathbb{N}\ [\varphi \mapsto N]\ M) \equiv \mathrm{hcomp}^i\ \mathbb{N}\ [\varphi \mapsto \mathrm{suc}(N)]\ \mathrm{suc}(M)$

A+B

$\mathrm{inl}(\mathrm{hcomp}^i\ A\ [\varphi \mapsto N]\ M) \equiv \mathrm{hcomp}^i\ (A+B)\ [\varphi \mapsto \mathrm{inl}(N)]\ \mathrm{inl}(M)$

$\mathrm{inr}(\mathrm{hcomp}^i\ B\ [\varphi \mapsto N]\ M) \equiv \mathrm{hcomp}^i\ (A+B)\ [\varphi \mapsto \mathrm{inr}(N)]\ \mathrm{inr}(M)$

**Cubical Agda has these rules*

- ✓ the unit
- ✓ functions
- ✓ pairs
- ✓ paths

negative types

- ✓ natural numbers
- ✓ disjoint sums
- ✓ the empty type
- ✓ the circle
- 🔥 universes

positive types